GPU-SPH simulation of Tsunami-like wave interaction
with a seawall associated with underwater disturbances

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Abstract
Investigation of the waves generated by underwater disturbances gives precious insight into the effect of man-made underwater explosions as well as natural phenomena, such as underwater volcanoes or oceanic meteor impact. On the other hand, prediction of the effects of such waves on the coastal installations and structures is required for preparation worthwhile criteria for coastal engineers to prepare a reliable design. This study aimed to investigate the interational effects of water waves generated by underwater disturbances on sea walls through numerical modeling using the Smoothed Particle Hydrodynamics (SPH) method. The simulation was performed using the Dual-SPHysics numerical code. Comparison of the numerical results with the experimental data extracted from case studies demonstrated the good capability of the SPH algorithm used in the numerical code in the simulation of initial wave generation by the underwater disturbance and its propagation over the body of water. To examine the wave force exerted on the walls, the results of laboratory experiences on the effect of tsunami waves on coastal structures were used to verify the numerical model. The study found that the phenomena with such nonlinear behavior can be very well simulated with a calibrated SPH model. We also explored the effects of this type of wave and temporal changes of its resultant force on the wall. In this article, the explosion-generated water wave produced much stronger fluctuations in the vicinity of the wall than did the solitary wave, thus naturally, it can be more destructive.

Keywords: Wave generation, Seawall, Numerical modeling, Wave force, Underwater explosion, Smoothed Particle Hydrodynamics (SPH), Graphics Processing Unit (GPU)

1. Introduction
The research on the likelihood and effects of underwater explosions is of particular importance for engineers in different fields of design or construction because of: (1) the development and presence of variety of marine installations and structures located in coastal regions, (2) the potential of manmade explosions near critical oil and gas assets in territorial waters of countries or explosions for construction purposes in shallow water, and (3) the possibility of explosion of offshore hydrocarbon facilities and pipelines. The review of research and experiences in the field of underwater disturbance demonstrates that the human quest to apply the waves generated by manmade explosions as an underwater disturbance generator since the nineteenth century. In the past century, the occurrence of unintentional and unexpected strong explosions in the oceanic area and

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related deep water tests and field experiments draw the attention of famous scientists and engineers working on marine research and wave theory to a new field of research focused on direct and indirect consequences of underwater explosions, and especially wave generation and propagation during this phenomenon (LeMe’haute’, 1971, 1976, 1993). In 1917, the explosion of Mont Blanc warship in the Bay of Halifax, Canada produced huge tidal surges that affected coastal areas and ports near the area. The destructive water waves generated during this accident had major implications for military and marine engineering. The number of deaths and the extent of damage to buildings and facilities following this accident were so high that it was labeled a human and national catastrophe (Greenberg and et al., 1993).

In 1944 and 1945, Professor Lich at Auckland University directed a series of field tests aimed at producing tsunami waves by underwater disturbances, as part of a common research operation called “Project Seal” involving New Zealand, US and British forces. The results of these surveys indicate that man-made explosions, produced in various depths of water, can generate waves of long wavelengths capable of destroying coastal areas. In the mid-decades of the twentieth century, many researchers, including Van Dorn (1961-1968), Whalin (1962-1970), and LeMehaute (1967-1998), used the results of underwater explosion field tests conducted by US Army to introduce and develop some theories in this regard.

Considering the difficulty and limitations of laboratory modeling and the hazards of field experiments on this subject, numerical simulation is the preferred method of marine engineers and researchers investigating the reaction of marine environments to underwater explosions. Other researchers, such as Crawford and Mader (1998) and Sims Skin and Fiske (2003) have employed the results and theoretical models of this field to develop a series of concepts for studying the effects of similar phenomena such as tsunami waves and waves generated by explosion of submarine volcanoes and for calculating the characteristics of waves generated by the collision of celestial objects and meteorites.

From the 1960s until the early 21st century, the scholars working in this field used to employ the Eulerian numerical methods to arrange governing equations into discrete form and solve them accordingly. But, because of reasons, such as the high velocity of the shock wave, emergence of large deformations, and separation of water particles from each other, solving the explosion-generated wave equations is a computationally complex process. With the development of meshless computational methods, improvement of numerical calculation techniques, and progress of hardware technology since the 1990s, the Lagrangian-based techniques have become the analysis method of choice in several fields. The most common application of such methods is in the simulation of shock propagation problems wherein variables change drastically with time.

One of the notable Lagrangian methods for solving fluid mechanics problems is the smoothed particle hydrodynamics (SPH) method. This method was developed by Monaghan and Gingold in 1977 to simulate problems in various physical aspects. The original application of this method was to analyze the problems concerning astronomy and movements of cosmic devices, but given the variety and complexity of particular problems of computational fluid mechanics, such as free surface problems in coastal engineering, and also because of the extreme difficulty of developing Eulerian models for these problems, SPH is being increasingly used for the analysis of marine phenomena.

(2013) tried to solve wave behavior and interaction by SPH method. Altomare et al. (2014) analyzed interaction of waves on armor type breakwaters. Altomare et al. (2015) investigated wave impact on coastal structures by SPH. Using parallel processing methods in combination with properties of graphical processing units of computers, a new generation of high performance computing techniques was developed to apply SPH method in CFD and oceanic computations. Domínguez et al. (2013), studied and developed new computational techniques for applying GPU to numerical modeling by SPH. Crespo et al. (2015) developed traditional SPH model on CPU into new form by applying GPU. Altomare et al. (2015) studied applicability of smoothed particle hydrodynamics for estimation of sea wave impact on coastal structures. Altomare et al. (2017), investigated modeling of long-crested wave generation and absorption by SPH on GPU.

Previous studies have shown that the use of this new methodology on GPU, provides better computational speed and precision to study complex marine phenomena that require long analysis time. Besides, for problems with complicated and large deformation on free water surface, a Lagrangian point of view is more practical and relevant.

In this study, we use the SPH method to simulate the waves generated by the underwater disturbance, such as explosion and examine its capability in this application. One of the main goals of this research is to study the impacting forces on the coastal wall in front of an explosion-generated water wave. Having the pressure distribution, the size of force applied to the wall could be easily determined. Naturally, knowing the load induced by explosion-generated water waves, engineers will be able to design the walls and coastal installations with better measures to counteract such effects. Another issue implicitly addressed in this study is the difference between the time histories of the force produced by the tsunami wave and the explosion-generated water wave on the vertical coastal walls.

2. Materials and Methods

The smoothed-particle hydrodynamics (SPH) method is a Lagrangian approach based on the simulation of the path of particle motion in an analytical domain. The major advantages of the SPH method are the independence of its simulations from the computational grid. This feature allows the SPH to bypass the limitation of the finite element or other mesh-based methods for computational problems and avoid the complexity imposed by re-meshing in the problems that require the simulation of particle segregation. Besides, with this numerical analysis method, problems of varying complexities can be numerically solved with significantly higher speed and reduced simulation run time.

Given the high complexity of calculations required for simulation of explosion-generated water waves, utilization of high-performance computing systems for this purpose is essential. For this purpose, we used a GPU-based parallel processing tool called DualSPHysics. The DualSPHysics is an open source numerical code capable of using GPU’s processing power, which has been developed in a joint research project involving several European universities. In this study, DualSPHysics and its capabilities in the Lagrangian reference frame were used for modeling, and the simulation results were compared with field and experimental data to ensure the validity of the numerical code. The hardware specifications of the simulation system are listed in Table (1).

The uncertainties that exist in field and laboratory observations of this phenomenon have caused numerous and complicated errors in the results. LeMehaute (1971) has said that the amount of laboratory errors is between 30% and 100%. To simulate this problem, a numerical model should be used to minimize the effects on the accuracy and results of such errors.

The SPH model has the ability to provide conditions for separating water particles from each other and take
Table 1: General Specification of the applied hardware and analytical parameters of various simulations

<table>
<thead>
<tr>
<th>Executive Parameters</th>
<th>Hardware Specifications Machine No. 1</th>
<th>Hardware Specifications Machine No. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle size (m)</td>
<td>10-3 ~ 5×10-1 CPU type AMD A8-3870 APU</td>
<td>CPU type Intel® Core™ i7-4700MQ</td>
</tr>
<tr>
<td>Total Particle</td>
<td>105 ~ 4×106 GPU type GeForce GTX 680</td>
<td>GPU type GeForce GTX 780M</td>
</tr>
<tr>
<td>GPU Run Time (sec)</td>
<td>1200 ~ 3400 CUDA Core No. 1536</td>
<td>CUDA Core No. 1536</td>
</tr>
<tr>
<td>CPU Run Time (sec)</td>
<td>3×103 ~ 40×103</td>
<td></td>
</tr>
</tbody>
</table>

into account the high displacement of the particles in the disturbance origin. Also, when waves impact on a coastal wall, it is possible to examine the collision of water particles with structures more precisely. These advantages make it possible to simulate the generation and propagation of waves from underwater explosion with adequate reality in comparison with the Eulerian type models that previously used by researchers (Mader 1998, Mader 2003, LeMehaute et al. 1986, LeMehaute et al. 1998). Also, by applying the parallel processing technique on GPUs, huge analyses could be performed with cheaper prices in a shorter computational time.

2.1. Hydrodynamics of explosion-generated water waves - variations of the initial conditions of the free surface

The governing equations for an underwater explosion consist of a set of conservation equations (mass continuity, momentum conservation, and energy conservation), the equation of state of detonation products, the equation of state of water, and the combustion equation, which is used to analyze the main process of an explosion in the water environment. For an explosion-generated surface wave to form, the gas contained bubble produced at the moment of explosion migrates toward water surface, and then, collides with the free surface, rupturing its wall. This is followed by the release of gas and fume constituents, which push the water body outward while throwing the water particles upward, thus generating the primary wave. Hence, the wave generation process can be divided into two phases. In the first phase, first, the nucleus of the bubble is formed, then, starts its evolution from a small high-pressure dome at the depth of water to a large bubble pulsating toward the free surface. The second phase starts when the bubble reaches the water surface, where it forms an initial deformation.

The initial deformation equation expressed by Eq.(1), which presents a second order parabolic curve, is based on the theoretical proposals of Poisson (1816) and Kranzer and Keller (1959):

\[ \eta(r) = \begin{cases} \eta_0 \left[ 2 \left( \frac{r}{R_0} \right)^2 - 1 \right] & ; \ r \leq R_0 \\ 0 & ; \ r > R_0 \end{cases} \]  

(1)

Where \( \eta_0 \) represents the maximum initial depth and the initial height of the edges of the explosion crater, \( R \) is the maximum span, \( "r" \) represents the distance to the center of the explosion location at the water level and \( \eta(r) \) is the function of water surface variations at \( "r" \). Normally, the characteristic parameters of crater shape, \( \eta_0 \) and \( R_0 \), can be evaluated by empirical equations or experimental results as a function of charge weight, water depth and depth of explosion (Van Dorn, 1961, LeMehaute et al. 1989). The initial geometry of the free surface as characterized by Eq.(1) is shown in Figure 1:
The field studies conducted by Penney (1945 and 1946) and Jordan (1965) have proven that only 40% of the energy released by an underwater explosion through the release of chemical energy stored in an explosive charge may generate water waves at free surface, as the rest of the explosion energy will be spent in generation of pressure waves in the water body and increase of water temperature. The effective part of the released chemical energy will be rapidly transformed into the potential energy stored in the form of the initial deformation of the water surface around the site of the explosion. This potential energy is the generator of base water wave.

Efficiency of an explosion as a wave producing mechanism is one of the important issues of the physical phenomenon. Amount of energy related to water waves produced by underwater disturbances was proposed by Whalin (1967) for different types of initial water surface deformation on the basis of Sakurai (1965) studies. The potential energy of deformed water elevation can be represented by Eq.(2):

$$E = \frac{\pi \rho g \eta_0^2 R_0^2}{6}$$ (3)

According to Figure 1, Van Dorn, LeMehaute and Hwang (1968) by performing physical laboratory tests show that for a parabolic crater, the radius of crater at edge of lip is related to the yield of the explosive charge by an empirical formula as per Eq.(4):

$$R_0 = 9.6W^{0.3}$$ (4)

Where the charge yield "W" has units of pounds of TNT, $R_0$ and $\eta_0$ have units of feet. $\eta_0$ can be determined for the parabolic crater by Eq.(5)

$$\eta_0 \times R_0 = 0.81H_{max} \times r$$ (5)

Which was resulted from experiments. $H_{max}$ is the maximum wave height in a wave group at a distance of $r$ from the location of the explosion. The value of $H_{max} \times r$ is dependent upon the depth of submergence of the charge and can be determined for any depth of submergence that was presented by Whalin, Pace and Lane (1970) using curve fitting of field data. At upper critical depth they proposed Eq.(6):

$$\frac{H_{max} \times r}{W^{0.54}} \approx 34$$ (6)

Also, LeMehaute (1980) presents slightly greater value by Eq.(7) on the basis of test results carried out at Waterways Experimental Station:

$$E = \frac{\pi \rho g \eta_0^2 R_0^2}{6}$$ (3)
In this study, equations 3 to 7 are used to calculate and align the energy of waves generated by an explosion with a solitary wave in the further studies.

2.2. Smoothed Particle Hydrodynamics (SPH) method

SPH is a Lagrangian method whereby the continuous medium can be discretized into a set of disordered points. SPH allows a function to be expressed in terms of its values at a set of particles by interpolation without needing any grid to calculate spatial derivatives. In this way, physical properties of each particle, such as acceleration, density, etc. are quantified as an interpolation of the same values in neighbor nodes. The SPH technique approximates a scalar function \( A(r) \) at any point with \( r \) vector of position, as follows:

\[
A(r) \approx \int A(r') W(r - r', h) \, dr'
\]

In Eq.(8), \( h \) is called the smoothing length and represents the influence of the nearest particles in a neighboring domain (see Figure 2), and it is therefore weighted in proportion to the distance between particles. Kernel function \( W \) is used to estimate the amount of participation through the parameter \( h \).

In Eq.(9), the summation is extended to all particles within the neighboring distance of particle "a", and the volume associated with the particle "b" is \( m_b \rho_b \), where \( m_b \) and \( \rho_b \) denote respectively the mass and the density of this neighbor particle. The kernel functions shall have several properties, including: positivity inside the area of interaction, compact support, normalization, and monotonic decrease with distance. Among notable options for the kernel is the quintic kernel proposed by Wendland (1995), where the weighting function vanishes for inter-particle distances greater than \( 2h \).

This kernel has been defined as (Altomare et al., 2014, 2015) by Eq.(10):

\[
W(q) = \alpha_D \left( 1 - \frac{q^4}{2} \right) (2q + 1) ; 0 \leq q \leq 2
\]

In Eq.(11), \( \dot{v}_a \) is the particle velocity, \( P \) is the particle pressure, \( g \) is the gravitational acceleration vector, and \( W_{ab} \) is the kernel function, which depends on the distance between particles "a" and "b". \( \Pi_{ab} \) is the viscous term according to the artificial viscosity definition proposed by Monaghan (1992):

\[
\Pi_{ab} = \left\{ \begin{array}{ll}
\frac{\alpha_v \cdot \dot{v}_{ab} \cdot \lambda_{ab}}{\rho_{ab}} & ; \quad \dot{v}_{ab} \cdot r_{ab} < 0 \\
0 & ; \quad \dot{v}_{ab} \cdot r_{ab} \geq 0
\end{array} \right.
\]
In Eq. (12), \( r_{ab} = r_a - r_b \) and \( v_{ab} = v_a - v_b \) are the particle position and velocity, respectively. \( \alpha_v \) is a coefficient that needs to be tuned to introduce the proper attenuation. In this study, we have used \( \alpha_v = 0.01 \), as it is the minimum value that prevents instability and spurious oscillations in the numerical scheme. 

\[
\bar{c}_{ab} = \frac{c_a + c_b}{2} \text{ is the mean speed of sound of particles,}\n\[ \bar{\rho}_{ab} = \frac{\rho_a + \rho_b}{2} \text{ is the mean density of particles and }\n\]

\[
\mu_{ab} = \frac{h}{r_{ab}} \cdot \frac{v_{ab}}{r_{ab} + \eta^2} \text{ where } \eta^2 = 0.01 \times h^2.
\]

The equation of states for ideal gases are:

\[
P = B \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \tag{13}
\]

\[
B = \frac{c_0^2 \cdot \rho_0}{\gamma} \tag{14}
\]

\[
c_0 = c(\rho_0) = \sqrt{\frac{\partial P}{\partial \rho}}_{\rho_0} \tag{15}
\]

Where \( B \) is a constant related to the fluid compressibility modulus, \( \rho_0 = 1000 \text{ kg/m}^3 \) is the reference density, chosen as the density at the free surface, \( \gamma \) is a constant, normally between 1 and 7, and \( c_0 \) is the speed of sound at the reference density.

3. Results

3.1 Simulation of Tsunami-like wave interaction with a seawall

Most of the previous researches about the effects of water waves generated by underwater explosion were focused on wave propagation and inundation effects. Besides, some scientists studied the near field or offshore effects of these waves on ships or underwater infrastructures. So far, very limited field or laboratory investigations have provided observational and qualitative information about the effects and magnitude of the impact of explosion-generated water waves on coastal structures. The lack of quantitative recorded data of interaction forces or pressure distribution on coastal structures due to underwater explosion induced wave, cause limitation for verification of numerical models of this phenomenon. Thus, in this article, before estimating the effects of these waves on coastal walls by numerical SPH analysis, the validity of the DualSPHysics code was checked with the help of variously available field datasets in different stages such as wave generation, propagation, and interaction.

First, the available experimental data about the generation of the initial wave by the underwater explosion and its propagation in the surrounding environment were used to verify the ability of the SPH method to simulate this part of the phenomenon.

In the second step, the ability of the numerical code to simulate the effects of the wave collision with the coastal structures was measured with the help of existing experimental data about the impulse applied by recorded laboratory data of artificially generated tsunami (solitary) wave on a vertical coastal wall.

In the final step, the water waves generated by the underwater explosion were simulated to estimate the force applied to the wall. On the other words, an explosion generated a wave with the equal energy of solitary waves of the second step was reproduced and simulated in the same numerical flume. The results of these waves with different types of origin are compared with each other.

In order to obtain the appropriate results in each of the models, the number of water particles was increased to the extent that the error results were less than 5% in two successive sequences of the numerical model (see tables 3,4,5). Subsequent simulations were executed in each case using the number of particles fixed for each example to assure the results.

3.1.1 Calibration and validation of the SPH Model for generation and propagation of explosion-generated water wave

We used two laboratory test cases to validate the DualSPHysics code and the SPH method incorporated therein. For generation of primary waves at the region
of explosion, we first calculate the amount of radius of the crater lip, $R_0$, by insertion of the charge weight "W" in the Eq.(4). Then, with the help of the range of expression $H_{max} \times r$ could be evaluated. At last, with the help of the Eq.(5) the amount of $\eta_0 \times R_0$ is calculated, and then $\eta_0$ may be estimated. In this way, the main parameters of Eq.(1) are determined and the deformed surface of water level as the initial boundary condition is applicable to the numerical model.

In order to measure the results of the numerical model in comparison with the experimental results, using the RMSE formula as Eq. (16), the output values of the numerical results were evaluated.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_m - X_e)^2}{n}}$$

where $X_m$ is numerical model output, $X_e$ is recorded data from experiments and $n$ is number of data.

In the following, we first describe experimental cases and then compare the results of numerical modeling with the experimental data:

A. The first set of experimental data is the result of studies conducted by Charlesworth (1945) at the London Road Research Laboratory. In this test, an underwater explosion was created in a 55 ft (16.76 m) wide and 70 ft (21.34 m) long rectangular gulf surrounded by the coast in the south and the west. The test area had an almost uniform water depth ranging from 15 to 18 feet (4.57 to 5.49 meters). The wave amplitude was recorded at a point 56.5 ft (17.22 m) away from the explosion at different times. A sample of the surface waves produced following the explosion of a 32-pound (14.5 kg) charge at a depth of 8 ft (2.44 m) was recorded. The initial condition of this case is considered on the basis of Eq.(1) as a parabolic crater with a lip. The characteristics values of this initial condition are: $\eta_0 = 2.07$ m, $R_0 = 8.28$ m. This initial crater is defined in the midpoint of the simulated basin. In Figure 3, the result of this test case is shown with the red curve. The results of the simulation with the SPH method were obtained by adjustment of different parameters for this sample.
In Figure 3, these results are plotted in the form of dash-line and dot-line curves. The range of variation of the parameters examined in this verification is presented in Table (2). The plotted results demonstrate the ability of SPH method to accurately simulate the initial waves generated by the explosion at the desired point. The maximum root mean square error (deviation of simulation results from the corresponding experimental values) was 11.4% and the minimum coefficient of correlation between simulation results and experimental data was estimated to 0.93.

Table 2. Characteristics of the main input parameters in the 3D SPH model used for simulation of the Charlesworth’s test (1945)

<table>
<thead>
<tr>
<th>Model Executive Parameters</th>
<th>Simulation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step Scheme</td>
<td>Symplectic</td>
</tr>
<tr>
<td>Kernel</td>
<td>Wendland</td>
</tr>
<tr>
<td>Viscosity Treatment</td>
<td>Artificial</td>
</tr>
<tr>
<td>Viscosity</td>
<td>0.01 ~ 0.25</td>
</tr>
<tr>
<td>Shepard Steps</td>
<td>20 ~ 30</td>
</tr>
<tr>
<td>No. Particles</td>
<td>1383984</td>
</tr>
<tr>
<td>Model Size (m)</td>
<td>100 × 100</td>
</tr>
<tr>
<td>CFL number</td>
<td>0.2</td>
</tr>
<tr>
<td>Coefficient of Sound</td>
<td>10 ~ 15</td>
</tr>
<tr>
<td>Smooting Coefficient</td>
<td>0.8 ~ 1.0</td>
</tr>
</tbody>
</table>

The second set of experimental data that was used to measure the efficiency of the SPH model is the data from the Prins’ experiments (1956 and 1957) in the Wave Research Laboratory at the UC Berkeley’s Engineering and Technology Research Institute. These experiments were carried out in a flume with a width of one foot (0.3048 m) and length of 60 feet (18.288 m). To produce an initial shock at water level (similar to an underwater explosion event), an air blowing-suction system was used to lower and raise the water level at the flume’s upstream. The initial water wave was released with a positive or negative height (compared to the flume’s normal water level) to generate a shock in the flume’s longitudinal direction. In this study, we used the test results reported for the uniform water depth of 2.3 ft (0.7 m), positive initial wave height of 0.4 feet (0.122 m), and length of 2 feet (0.61 m) at the left side of the numerical flume. The initial wave is considered at the center of the basin model. The changes in the water elevation were measured at a point 15 ft (4.57 m) away from the shock initiation point. In Figure 4, these measurements are displayed by the blue curve. In Figure 4, the results of the simulation with the SPH method following the adjustment of different parameters for this test are shown in the form of dash-line and dot-line curves. The range of variation of the parameters examined in this verification is presented in Table (3). As the presented results indicate, the SPH method can accurately simulate the initial waves generated by the explosion at the desired point.

In this case, the maximum root mean square error (deviation of simulation results from the corresponding experimental values) was 12.4% and the minimum coefficient of correlation between simulation and experimental results was 0.89.
3.1.2 Calibration and validation of the SPH Model for estimation of laboratory generated tsunami (solitary) wave induced force on the coastal wall

A large number of experiments have been carried out to measure the force exerted on the sea walls by tsunami waves. In this part of the study, we use two laboratory experiences to assess the capability of the SPH model for estimating the wave force on the coastal walls.

For this part of the study, the amount of energy for solitary waves is also calculated, so that in the next...
section of the paper, the surface waves due to underwater explosion with their equivalent energy are generated.

Longuet-Higgins (1974), reviewed a number of successful models for prediction of solitary waves. His studies showed that using the Eq.(17) may represent excellent results to predict the waveform.

\[
\eta = Ae^{-\mu|x|} + Be^{-\nu|x|} \quad \frac{\eta}{A} = 1.5389 \quad \frac{\eta}{B} = -0.7093 \quad \frac{\mu}{\nu} = 1.0495 \quad \frac{\nu}{\mu} = 1.4630
\]  

(17)

He also assumed the equations 18 and 19 to calculate the kinetic "T" and potential "V" energy of the solitary wave. Then total energy "E" of wave could be calculated by \( E = T + V \).

\[
T = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2} \left( u^2 + v^2 \right) \eta \, dy \, dx
\]  

(18)

\[
V = \int_{-\infty}^{\infty} \frac{1}{2} g \eta^2 dx
\]  

(19)

Where \( u \) and \( v \) are velocity component of water particle in horizontal and vertical directions, respectively. Also \( \eta \) characterizes water elevation and \( h \) is water depth.

By applying equation 17 into equations 18 and 19, he was able to provide a very good approximation for estimating the energy values of the solitary wave, which is considered in equations 20 and 21, properly.

\[
T = 0.527 \, gh^3
\]  

(20)

\[
V = 0.431 \, gh^3
\]  

(21)

Finally, the total energy of a solitary wave can be approximated by Eq.(22) as follows:

\[
E = 0.958 \, gh^3
\]  

(22)

Besides, in this section, the corresponding simulation results obtained with the DualSPHysics numerical code are presented correspondingly for each case.

A. The results of laboratory-based physical modeling performed by Robertson et al. (2011). This test set was constructed in a laboratory flume with a length of 83.13 m, the width of 3.66 m and depth of 4.57 m in the laboratory of Oregon State University (see Figs. 5 and 6).

![Fig. 5: Side view of the Laboratory Flume of Robertson et al. (2011) at the University of Oregon State Laboratory](image)

![Fig. 6: SPH environment of Robertson et al. Test case flume](image)
To reproduce the experimental data in simulation, a solitary wave (representing the tsunami wave) with a wave height of 1.064 meters and wavelength of 30 meters was generated in the water depth of 2.66 m. The propagation of the generated solitary wave in the numerical simulation of experiments in deep water region is displayed in Figure 7. Time history variations of pressure distribution on a vertical wall are shown in Figure 8. In Figure 9, time history variations of the force applied to the wall are compared with the corresponding experimental data. Also, variations of impulse with time are plotted in Figure 10. The maximum sum of squares of relative errors of simulation is 5.03% for force values and 1.41% for impulse values. The minimum coefficient of correlation between simulation results and experimental data is 98.16 for force values and 99.96 for impulse. The total energy of this wave is estimated to be $E \approx 177$ kJouls.

Fig. 7: Snapshots of solitary wave generation (by paddle) and propagation along the numerical SPH flume for Robertson’s experiments
Fig. 8: Time history of pressure distribution on the vertical wall

Fig. 9: Time history of the force on the wall during the wave impact to the wall, (a) laboratory (circle) and (b) simulation of the SPH (star)
Fig. 10: Time history of the impact on the wall by Robertson (2011), laboratory results (dash) and SPH results (line)

B. Results of laboratory experiments performed by Yu Yao et al. (2017)

This case study was carried out in a laboratory flume with a length of 36 m, the width of 0.55 m and depth of 0.60 m in Nanyang Technological University, Singapore. A 1:5.7 steep fore slope starting from 16.33 m away from the wave paddle was constructed and a rectangular seawall model with 6 cm long, 5.5 cm wide, and 5 cm high, was placed on the top point of the slope to represent a vertical caisson type seawall (see Figure 11).

The recorded data by G1 and G2 wave gauges, which were located 5.24 m and 0.14 m upstream from the seaside of the wall, respectively. Besides, induced dynamic pressure by wave impinging on the wall was recorded by a pressure sensor installed on the center of the seawall (see Figure 12).

To reproduce the experimental data in the simulation, a solitary wave with a wave height of 0.05 m and wavelength of 2.05 m was generated in the water depth of 0.36 m. The numerical model is considered symmetrical with sufficient far sloped beach from point of initial wave generation in the middle of the numerical flume for canceling of reflecting waves. Geometry and initial generated solitary wave in the numerical flume are presented in Figures 13 and 14.

Fig. 11 Sketch of the experimental setup (reported by Yu Yao et al., 2017)
Fig. 12: Seawall model and pressure gauge location on the wall (reported by Yu Yao et al., 2017)

Fig. 13: SPH environment of Yu Yao et al. Test case flume (up) and initial solitary wave generated in the model (down)

Fig. 14: Enlargement of solitary wave generated in SPH model (height of generated wave is approximately 5 cm)
Comparing of measured free surface elevations with the numerical model at different wave sampling locations (by G1 and G2 sensors) are presented in Figures 15 and 16. In addition time history of pressure at the midpoint of the seawall is plotted in Figure 17. Both of free surface and pressure graphs are presented in non-dimensional scale as per following pictures and show good agreement between experimental and SPH results. The total energy of this wave is estimated to be $E \approx 0.44$ kJouls.
3.1.3 Simulation of the force applied by interaction of explosion-generated water wave and wall

As mentioned earlier, in this section, we characterize the surface wave produced by a fictitious explosion with equivalent energy to the solitary waves of Section (3.1.2) in the numerical flume. We compared the characteristics of wave propagation and interaction force with previous results in order to measure and describe variations in the amount of force on the wall and water level in similar locations. For quantifying the energy of reproduced tsunami-like waves due to explosion, by referring to Eq. (3) the value of $\eta_0 \times R_0$ should be estimated for each case. Then, by use of Eq. (5) simply $H_{\text{max}} \times r$ could be evaluated and if this value be inserted in the equations 6 and 7, the margin of charge weight "W" may be calculated. At last, by applying charge weight value into Eq. (4), the radius of crater at edge of lip $R_0$ and then $\eta_0$ are found easily (see Table 4). Now the initial boundary condition of water free surface may be defined by Eq. (1) and considered in numerical model.

For applying this procedure to the laboratory setup which was performed by Yu Yao et al., an explosion is assumed at the middle of the flume that produces a wave with crater depth and lip height of $\eta_0 = 0.23 \text{ m}$ and crater span of $R_0 = 1.3 \text{ m}$ at the maximum height of the lips, which propagates upstream and downstream along the flume’s length (see Figures 18 and 19). All other properties such as the geometry of flume, water depth, etc. are considered same as previous.

<table>
<thead>
<tr>
<th>Case Study</th>
<th>Energy (kJouls)</th>
<th>$\eta_0 \times R_0$ (m$^2$) Eq. (3)</th>
<th>$H_{\text{max}} \times r$ (m$^2$) Eq. (5)</th>
<th>Average Charge Weight (kg)</th>
<th>$R_0$ (m) Eq. (4)</th>
<th>$\eta_0$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Robertson et al. (2011)</td>
<td>177</td>
<td>5.9</td>
<td>7.25</td>
<td>3.83</td>
<td>5.4</td>
<td>1.5</td>
</tr>
<tr>
<td>B Yu Yao et al. (2017)</td>
<td>0.44</td>
<td>0.6</td>
<td>0.75</td>
<td>0.03</td>
<td>1.3</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Fig. 18: The Initial geometry of the explosion generated water wave for Yu Yao et al. Test case

Fig. 19: Enlargement of primary surface wave generated by underwater explosion in SPH model for Yu Yao et al. Test case
Figure 20 shows the time history of water level variations at two gauges in deep and shallow regions of flume. Figures 21 and 22 display the time history of force on the vertical wall and the snap shots of wave location during the insertion of peak forces at 25-meter and 50-meter, respectively. The major modeling and simulation parameters are listed and described in Table (5).
Fig. 22: Snap shots of wave impact on wall at (a) t=6.74 sec, (b) t=7.5 sec, (c) t= 8.78 sec

Table 5: Characteristics of the main input parameters in the 2D SPH model used for comparison of simulations

<table>
<thead>
<tr>
<th>Modeling Parameters</th>
<th>Executive Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle size (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>Symplectic</td>
<td></td>
</tr>
<tr>
<td>Time step algorithm</td>
<td></td>
</tr>
<tr>
<td>Wendland</td>
<td></td>
</tr>
<tr>
<td>Model overall size (m)</td>
<td>200</td>
</tr>
<tr>
<td>Kernel type</td>
<td></td>
</tr>
<tr>
<td>CPU run time (sec.)</td>
<td>0.2</td>
</tr>
<tr>
<td>GPU run time (sec.)</td>
<td></td>
</tr>
<tr>
<td>Total Number of Particles</td>
<td></td>
</tr>
<tr>
<td>Tsunami model</td>
<td>46702</td>
</tr>
<tr>
<td>Explosion model</td>
<td>45374</td>
</tr>
<tr>
<td>Tsunami model</td>
<td>3062</td>
</tr>
<tr>
<td>Explosion model</td>
<td>3560</td>
</tr>
</tbody>
</table>

According to the plotted water level variation diagrams, solitary wave resulted in conservation of depth at the points close to the wave initiation point, but explosion-generated water wave led to depth reduction at these points. In other words, by moving away from the source of the wave generator, the explosion waves which was simulated by an initial parabolic water level, in comparison with a solitary wave as a representative of the tsunami wave, show attenuation of wave amplitude and force, when they are propagated on a flatbed.

4. Conclusions

In this research, we used the DualSPHysics numerical code to simulate and examine the forces induced by the waves generated by an underwater explosion on a vertical coastal wall. To study the behavior and effects of the explosion-generated water wave, the results were investigated in comparison with the effects of a solitary wave of equal energy. The following conclusions are deduced from the results:
(a) The validation results show the ability of the DualSPHysics code and the SPH method to accurately simulate the free surface variations due to an underwater explosion. In this study, we used two sets of laboratory data to assess the ability of the SPH method in the simulation of phenomena for which recorded field or laboratory data have variations and inherently these types of experiments may record with large error tolerances. As shown in the validation part of the study, the matching of numerical simulation results with laboratory values demonstrated the ability of the numerical model to capture the phenomenon with ill-conditioned data.

(b) Although the SPH model used in this study has an accuracy order of $1 \leq O(r) \leq 2$, which is more inclined toward 1, the model verification phase revealed the ability of this method to predict a complicated nonlinear phenomenon of high complexity with proper precision. On the other hand, this method can reproduce physical characteristics of complex CFD problems in a short analysis time, which is a valuable benefit for engineers to understand the impacts of such a dangerous condition for coastal structures.

(c) Due to the behavior of the explosion-generated water waves, caused by the lowering of the free surface of the water at the “ground zero” (i.e. the location of the center of an explosion), the water elevation was decreased and then increased by the progressive negative wave in the vicinity of the obstacle. These statuses produced reduction and increase of pressure and force on the wall respectively.

(d) In fact, this effect is different from the forces resulting from the collision of the tsunami waves (solitary) on the coastal walls, in the opposite way. Thus, in identifying the risk of surface waves generated by underwater explosion than the tsunami waves, it can be inferred that this type of waves initially show a decrease in water level, resulting in a reduction of pressure and force on the wall, and then these values are an uptrend.

(e) Obviously, both time history of forces have approximately the same peak value, but total impact time for tsunami waves is longer than those related to explosion generated waves. This result shows that the explosion-induced waves are very dissipative and their effects are important for near field objects.

(f) The force exerted on the vertical wall by explosion-generated water waves has a periodic nature and attenuates with time. In contrast, the force of a solitary wave with an equal peak acts on the structure in a short time period and then settles down.

(g) The explosion-generated water wave has a stronger perturbation character and remains imposed on the structure for a longer period of time. Obviously, repeating the simulation by taking into account the effect of water particles released into the surrounding area and their return to the water surface (which may produce secondary waves) will result in longer duration of turbulence in the water level and more fluctuations in the force exerted on the wall.

(h) For the 2-D models, utilization of GPU-based method yields a better computational speed than the parallel computation in multi-core CPU. In the case of this study, using the former approach reduced the computational time by at least 60%. This feature could be particularly advantageous for the processing of more complex models, especially those involving the effects of sudden and impulsive loads, such as explosion generated water waves, and save the engineers precious time in design as well as damage estimation.
(i) For the 3-D models, the use of GPU-based method led to 75-90% reduction in the execution time, which demonstrates the greater efficiency of simulation with GPU and the SPH technique for more complex problems. This significant reduction of run time also signifies the nonlinear relationship of processing time with the model dimensions and the effect of large scale of analyses that have a greater number of particles on the computational time and the performance of the optimization simulation algorithm in the SPH method.

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