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Least Squares Analysis of Noise-Free Tides Using Energy Conservation and Relative Concentration of Periods Criteria

Mahmoudof, Masoud[1*]; Banijamali, Babak[2]; Chegini, Vahid[1]

1- Iranian National Institute for Oceanography, Tehran, IR Iran

2- Darya-Bandar Consulting Engineering (DBC), Tehran, IR Iran.

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Abstract

In this article, a new approach to tidal harmonic analysis is introduced. This approach incorporates more constituents in the least squares method for a fixed duration of noise-free tidal record and results in a more accurate tidal prediction. Moreover, it is demonstrated that 135 days of hourly data, which is significantly less than 369 days data in the Rayleigh criterion, is sufficient for the analysis of 68 main tidal constituents with excellent accuracy. Simultaneous variations of tidal constituents amplitudes are studied while the accuracy of the Rayleigh criterion is evaluated. It is observed that a more reliable criterion for inclusion or omission of tidal constituents in the least squares analysis instead of the Rayleigh criterion may be strived for, since the latter solely estimates the mutual effects of two neighboring constituents while neglecting the collective and combined effects of all constituents. Hence, an energy criterion as well as an allowable error value were introduced in the quest for better accuracy. Using this concept and considering the combined effects of all pertinent constituents, a novel approach is developed enabling the proper selection of tidal constituents for the least squares method of tidal analysis. According to this new approach, the underlying constituents are selected considering the time-span of the tidal time-series record, the tidal potential amplitude coefficients of the constituents, a concentration coefficient and the standard deviation of all constituents periods.

Keywords: *Tidal analysis. Lead time. Energy criterion. Periods' concentration*

1. Introduction

This research was undertaken to devise a more efficient tidal analysis method capable of predicting tides more accurately for longer-term requiring the least possible duration of tidal data. Nowadays, there is a quest by physical oceanographers and coastal engineers to provide more accurate methods for tidal analysis and prediction, particularly, the objective of arriving at increased accuracy for long-term tidal prediction (Emery and Thomson, 2001). The preciseness of tidal constituents analysis is the pre-requisite of an accurate tidal prediction, therefore, the reliability of results is governed by the time-span of the tidal time-series and hence, the maximum number of tidal constituents which may be taken into consideration (Reddy, 2001). Nevertheless, the installation of a permanent tidal station in many places around the globe may not be possible due to the prohibitive associated costs among other possible reasons. In brief, tides may be considered

* *E-mail: m_mahmoudov@inio.ac.ir*

as the result of interactions by some tidal constituents which are rooted in astronomical phenomena, where the relative importance of each constituent may be represented by its tidal potential amplitude coefficient.

The least squares method has been conventionally utilized for the harmonic analysis of tides, as far back as by Horn, in 1960 (Stewart, 2007). Using this method, where water-level variations data are analyzed with respect to some specific constituents, it is well-known that an insufficient time-span of water level data may result in an ill-conditioned and unstable set of equations.

While the existence of errors in tidal analysis and prediction, stemming from instrumentation and the inherent limitations of the particular analysis, are a fact of life, however, it is the underlying aim of tidal analysis and prediction models to quantify threat and minimize any such errors.

It deserves mentioning that the selection of harmonic constituents for a fixed time span of data yields some diversity in the least squares treatment of tidal analysis. For instance, the Foreman model (Foreman, 1977) is widely employed around the world where the procedure consists of selecting constituents based on an expected relative importance as well as a frequency separation between two neighboring constituents.

Notably, with computer hardware advances of today, all main and shallow water tidal constituents, amounting to more than 500, may be included in computational processes, such as Leffler and Jay, 2008. In this article, it is attempted to present a new and more accurate method to include the maximum number of possible tidal constituents while minimizing the prediction error, thereby, particularly, serving the practical need in regions of limited tidal data around the world.

2. Methodology

2.1. A Brief Recap of the Least Squares Method of Tidal Analysis

Assuming noise-free sea water-level variations are formed by *M* tidal harmonic constituents, equation (1) formulates the mathematical function of water-level variations with respect to tidal constituents (Emery and Thomson, 2001; Foreman, 1977), as follows:

$$y(t) = a_0 + \sum_{j=1}^{M} a_j \cos 2\pi(\sigma_j t_n - \varphi_j) \tag{1}$$

where in the above equation, a_0 is the mean value of the record, expressed relative to a chart datum level, per definition $t_n = n\Delta t$; while a_j, σ_j and φ_j are respectively the constant amplitude, frequency and phase associated with the *j*th constituent. The least squares analysis is based on achieving maximum conformity between theoretical and field record values.

Following the conventional least squares method, each constituent is introduced by two sine (S_j) and cosine (C_j) expressions, as elucidated by Foreman (1977). Upon solving the ensuing matrix of equations, values of A_j and φ_j may be calculated according to the following equations (2) and (3):

$$a_j = (C_j^2 + S_j^2)^{(1/2)} \tag{2}$$

$$2\pi\varphi_j = \tan^{-1}\frac{S_j}{C_j} \tag{3}$$

2.2. Methodology of the Proposed Alternative Approach

2.2.1. Introduction of an Energy Criterion

For the purposes of this study, a FORTRAN code was prepared to incorporate the least squares method enabling the assessment of varying tidal constituent characteristics. The software includes a prediction capability based on equation (1) as well as the least squares method, in a manner such that cross-checking the analysis results versus the predictions becomes possible.

In continuation, a sensitivity analysis of the prediction accuracy versus the duration of the record has been performed for a case consisting of 10 tidal constituents, with a_0 being equal to 2.1 meters, with the pertinent artificial characteristics being as shown in Table1, as follows:

Table 1. Artificial Characteristics of 10 Constituents

Constituent name	M2	S2	K1	P1	K2	NO1	R2	O1	H2	T2
Period (h)	12.4206	12.00	23.9344	24.0658	11.9672	24.8332	11.9835	25.8193	12.4030	12.0164
Amplitude (m)	1.05	0.45	0.23	0.15	0.12	0.09	0.10	0.08	0.06	0.13
Phase (deg)	23	142	196	326	251	14	287	96	130	205

Firstly, the initial water-level time-series were generated for a two months period, consisting of *1,440* water-level values with a constant one hour interval, in line with the common practice for the mentioned interval in tidal analysis (Emery and Thomson, 2001). Currently, it is customary to utilize the Rayleigh criterion in tidal analysis. In the quest for an alternative criterion to Rayleigh criterion, it is notable that tides being classified as long waves and thereby treatable by basic wave theories, adhere to the hypothesis that the total energy of each tidal constituent is proportional to the square of that constituent's amplitude (Sorensen, 1997), as in equation (4) below:

$$E_i \propto a_i^2 \tag{4}$$

For a wave consisting of linear superposition of sinusoidal constituents of varying amplitudes, a_j, the total wave energy in a linear sense is related to the sum of the square of amplitudes, as described in equation(5), below:

$$E \propto \sum a_j^{\,2} \tag{5}$$

Hence, the sum of the amplitudes squared, including the tidal constituents, may be considered as a main criterion within the least squared analysis. This criterion, hereforth termed the energy criterion, with its parameter as denoted by *A*, given in unit of m^2, may be stated as follows:

$$A = \sum a_j^{\,2} \tag{6}$$

For illustration purposes, the actual value of this criterion per the least squares method for the previous example with *10* supposed tidal constituents is *1.4398* m^2 whereas, its values for *100*, *200* hours and one month subsets of the initial base time-series become *4.433*, *1.759* and *1.612* m^2, respectively. Furthermore, this parameter obtained from a Fourier analysis considering the full *2* months long time span is *1.444* m^2.

To investigate the fluctuation of this parameter over the duration, a continuous least squares analysis for up to *120* days was performed and the variation of energy criterion was determined, as in figure(1), below:

Clearly, the fluctuation of *A* is very intense initially times, however, when about *1200* hours have passed, an almost steady state is reached.

To complement the introduction of *A* as a certain numeral criterion, the relative error may be quantified. For practical purposes, an upper limit of 1% on the relative error is deemed allowable in a conventional sense, allowing the definition of the lead time as the onset of reaching the allowable error limit.

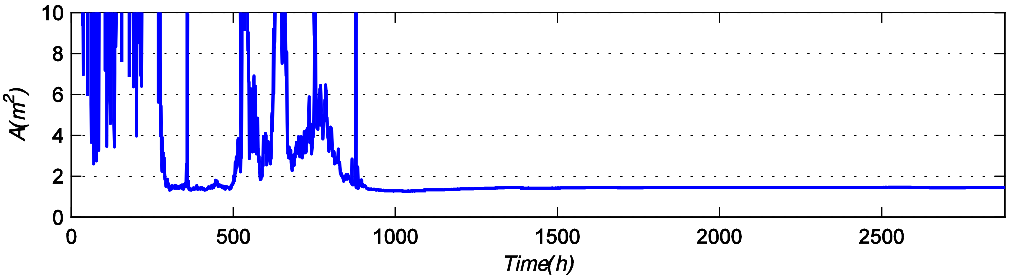

Fig. 1: Continuous Variation of the Energy Criterion from the Least Squares Analysis of Tides up to 120 Days

2.2.2. Reviewing the Rayleigh Criterion using the Energy Criterion

Conventionally, tidal constituents are selected based on the Rayleigh criterion for the least squares analysis in many practical tidal models. If F_1 represents the first constituent's frequency to be included in the least squares analysis, the second constituent with F_2 frequency, as termed the Rayleigh comparison constituent, will be included based on the comparison as stated via equation 7, as follows:

$$\left|F_1 - F_2\right| T_t \geq RAY \tag{7}$$

RAY is commonly given the value of 1 in the absence of background noise, however, it may be specified differently, and T_t stands for the time span of the proposed record to be analyzed (Foreman, 2001) where, it is also named as the time separation of the second constituent from the first (Emery and Thomson, 2001; Pugh, 1987). If equation (7) is satisfied then, the second constituent would be included otherwise, it would be eliminated from the analysis. In fact, the time span of the data record based on the Rayleigh criterion determines whether or not the second neighboring constituent will be included (Emery and Thomson, 2001). To compare the Rayleigh and Energy criteria, a numerical assessment was undertaken for the case with the characteristics of 4 neighboring hypothetical tidal constituents being as presented in Table (2) below:

Table 2. Characteristics of Four Hypothetical Constituents

Constituent's name	Period (h)	Amplitude (m)	Phase (deg)
A	12.10	1.5	45
B	12.05	1.2	165
C	12.00	1.0	185
D	11.95	0.8	265

Based on the energy criterion, the lead time was calculated for the *A* and *B* constituents. Then, the same process was executed for first three and all four constituents. The lead time based on the energy criterion was calculated for the first, second and third case to be *22*, *218* and *526* hours, respectively. Notably, based on the Rayleigh criterion the time separation of *A* and *B* constituents was greater than other pairs of frequencies, being calculated according the Rayleigh criterion from equation (8):

$$\left|\frac{1}{T_B} - \frac{1}{T_A}\right| T_t \geq 1 \Rightarrow \left|0.082987 - 0.082644\right|$$
$$T_t \geq 1 \Rightarrow T_t \geq 2916 \ h \tag{8}$$

The results of this comparison are summarized in Table (3):

Table 3. Comparison Stability Time for Rayleigh and Energy Criterions

Combinations	Required time based on the Rayleigh criterion (h)	Required time based on the Energy criterion (h)
A, B	2916	22
A, B, C	2916	218
A, B, C, D	2916	526

From this case study, it follows that the Rayleigh criterion considers two neighboring constituents in contrast to taking overall collective effects of a number of neighboring constituents. Thus, the case of using the Rayleigh criterion for tidal least squares analysis in lieu of the lack of long time water level data, which is the case in many places around the world, would result in the elimination of a considerable number of tidal constituents leading to significant errors of longer-term tidal prediction.

2.2.3. Evaluation of the Lead Time using the Energy Criterion and Concentration of Periods

Another case study was undertaken to study the lead time using a statistical approach, for a case consisting of nine neighboring hypothetical tidal constituents. The first and the last periods were selected as 11.85 h and 12.15 h, with the remaining seven constituents being variable in a symmetric sense between these two periods with a constant average of 12.00 h. The time intervals between the periods were set symmetric in order to keep a constant average for controlling all effective factors. Subsequently, the lead time was calculated for all constituent groups considering the 1% allowable error limit. Table (4) encapsulates the obtained results for the lead time.

Table 4. Obtained lead Time values for a hypothetical case using the Energy Criterion Approach

Cons no* / Analysis no	Periods (h) 1	2	3	4	5	6	7	8	9	Lead time (h)
1	11.85	11.89	11.92	11.96	12.00	12.04	12.08	12.11	12.15	**1948**
2	11.85	11.88	11.91	11.96	12.00	12.04	12.09	12.12	12.15	**1788**
3	11.85	11.86	11.88	11.90	12.00	12.10	12.12	12.14	12.15	**1878**
4	11.85	11.86	11.87	11.88	12.00	12.12	12.13	12.14	12.15	**2244**
5	11.85	11.95	11.97	11.99	12.00	12.01	12.03	12.05	12.15	**4483**
6	11.85	11.97	11.98	11.99	12.00	12.01	12.02	12.03	12.15	**6186**

*Constituent number

Evidently, as the concentration of periods increases, the lead time will tend to grow. If the concentration happens near the boundaries of the time-span, the lead time will grow considerably; however, when the concentration happens near the center of time-span, the lead time will grow very considerably. So, the relative concentration coefficient should be defined and taken into account in the pertinent method.

A recommended relative concentration coefficient, γ, is proposed in equation (9):

$$\gamma = \frac{\sum_{2}^{n-1}\left((x_i - x_{i-1})^2 + (x_{i+1} - x_i)^2\right)}{(x_n - x_1)^2} \qquad (9)$$

where the value of γ varies between *0.5* to *2.0*. If the constituent periods are set uniformly, then the value of γ will amount to $\frac{2(n-2)}{(n-1)^2}$

As a result, the statistical picture of Table (4) was accomplished in detail using nine neighboring constituents and periods from 11.85 h to 12.15 h, with 0.05 h intervals. For each group of constituents, the relative concentration coefficient was calculated and stability time was investigated. Figure 2 shows the diagram of the lead time versus the relative concentration coefficient.

As observable from figure 2, the lead time behavior is inclined toward the red vector whenever the periods concentrate with respect to the midpoint of the group, while it tends toward the green vector when they concentrated toward boundaries of the span.

A closer examination of figure 2 yields that a possible difference between two tendencies represented by the vectors could be rooted in the standard deviations of the groups. Therefore, the standard deviation was calculated for each particular group, elucidating that whenever the concentration of periods tends from the midpoint toward boundaries in a symmetrical sense, then the standard deviation values rotate from the steeper red to the milder sloping green vector in a clockwise sense around the point of intersection of the two vectors.

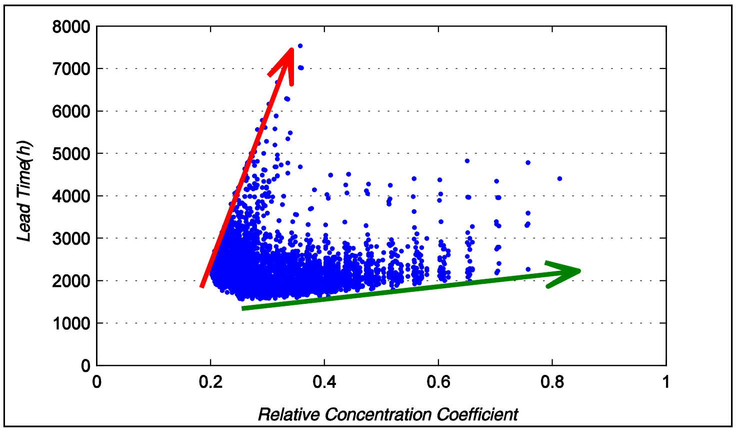

Fig. 2: Variation of Stability Time with respect to Relative Concentration Coefficient

For better illustration, a two-dimensional plot is presented in figure 3. The lead time could be estimated utilizing this figure after setting the parameter values for the relative concentration and the standard deviation of the group periods, with the latter being normalized through dividing it by the length span of the periods.

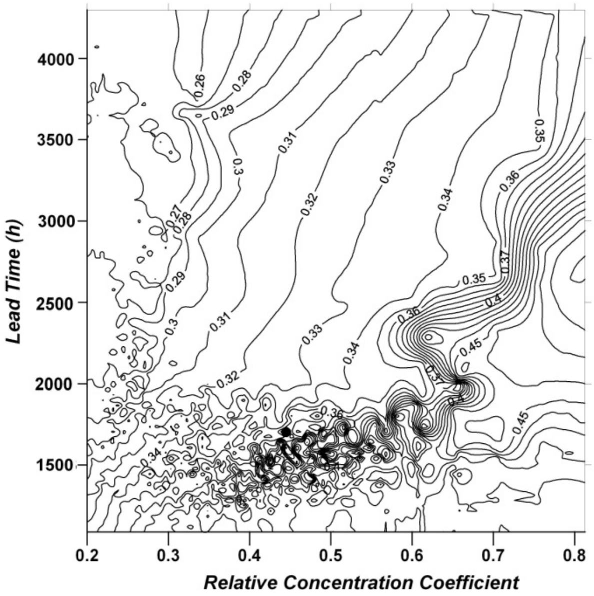

Figure3. Variation of Stability Time with Respect to Relative Concentrated Coefficient and Standard Deviation and Normalized Standard Deviation

In the example used for illustrating the new approach, the normalized standard deviation varied from 0.25 up to 0.5, but generally, its value behaved according to the following equation (10) whenever the periods concentrated near the midpoint of the span:

$$S_n = \frac{S}{l} = \frac{\sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}}{l} = \frac{\sqrt{\frac{2}{n-1} * \frac{l}{2}}}{l} = \sqrt{\frac{1}{2(n-1)}} \qquad (10)$$

Furthermore, this value tends toward 0.5 according to equation (11) whenever the periods concentrate symmetrically near boundaries of the span.

$$S_n = \frac{S}{l} = \frac{\sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}}{l} = \frac{\sqrt{\frac{(n-1) * l^2 / 4}{n-1}}}{l} = 0.5 \qquad (11)$$

The spectrum associated with figure 3 is depicted in figure 4:

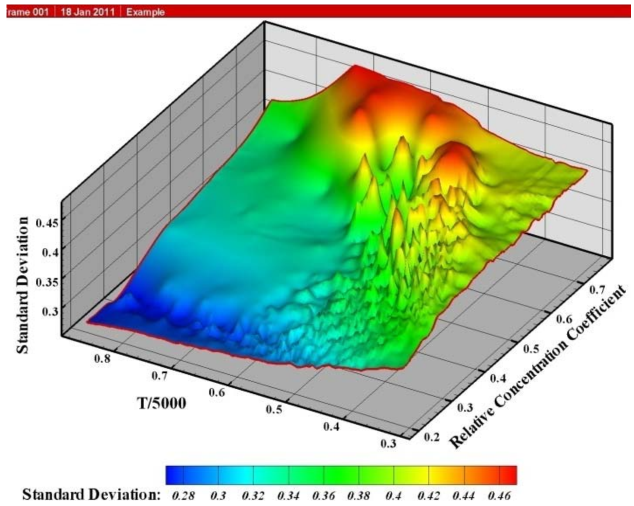

Fig. 4: The Spectrum of Stability Time

Having investigated the relationship between the stability time, the relative concentration coefficient, and the standard deviation, it remains to explore the sensitivity of the lead time as a function of other interplaying factors, such as the number of constituents, average of periods corresponding to all constituents in the group, and the length of the overall range of periods, according to scenarios composed of seven groups of constituents, with three up to nine members of equidistant intervals, whereas the median was set at *12.00* h. The lead time was investigated for these cases.

This assessment was performed on groups by the same numbers of members and different average of periods of *6.00*, *10.00*, *15.00*, *18.00* and *24.00* h. The variation of the lead time versus the number of group members is shown in figure 5 for the mentioned *42* groups.

As can be observed, after smoothing and interpolation of the curves a power function, as defined by: $T_t = \alpha (n-2)^2$, shows acceptable accuracy, where in the latter n stands for the number of group members and α is a constant coefficient.

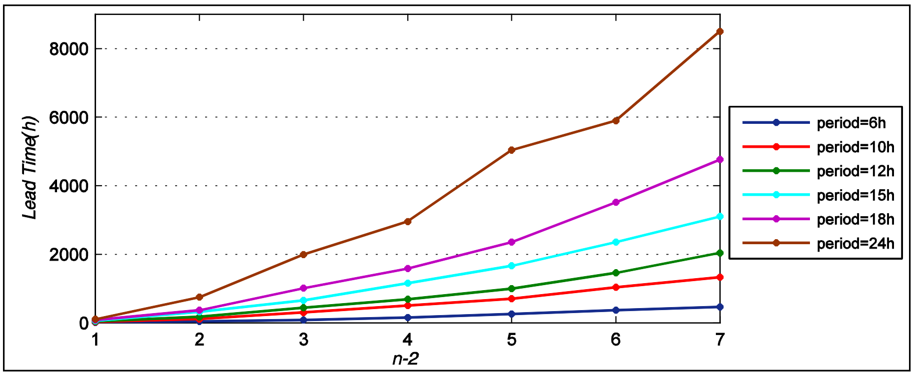

Fig. 5: Relationship of the Stability Time as a function of the Number of Tidal Constituents for constant Average of Period values

Therefore, the ratio of the lead times for two constituents groups, η, with equal period averages and period spans may be calculated as via equation (12):

$$\eta = \frac{T_{n1}}{T_{n2}} = \left(\frac{n_1 - 2}{n_2 - 2}\right)^2 \tag{12}$$

where T_{n1} is the stability time for the group with n_1 members, and T_{n2} the stability time for the group with n_2 members.

If the last result for 42 groups is divided by the average of periods of the groups, the figure 6 is derived. Clearly, a power function would be suited to such interpolation with the equation $T = \alpha\tau^2$ providing an excellent fit. Therefore, the ratio of the lead time for two constituents groups with equal number of group members and period spans and different period averages, denoted by μ, may be derived from equation (13).

$$\mu = \frac{T_{m1}}{T_{m2}} = \left(\frac{\tau_1}{\tau_2}\right)^2 \tag{13}$$

where T_{m1} the stability time of the group with τ_1 average and likewise, T_{m2} the lead time of the group with τ_2 average.

Additionally, to estimate the effects of the period span on lead time, a hypothetical group of 9 constituents periods with equal intervals ranging from 11.85 up to 12.15h was investigated. The lower and upper limits of the span were varied symmetrically. The intervals of periods varied on the same scale. For each group, the lead time was investigated. The same analysis was performed for groups by the same number of members and different period average values of 6.00, 10.00, 15.00, 18.00 and 24.00h, with the results as presented in figure 7.

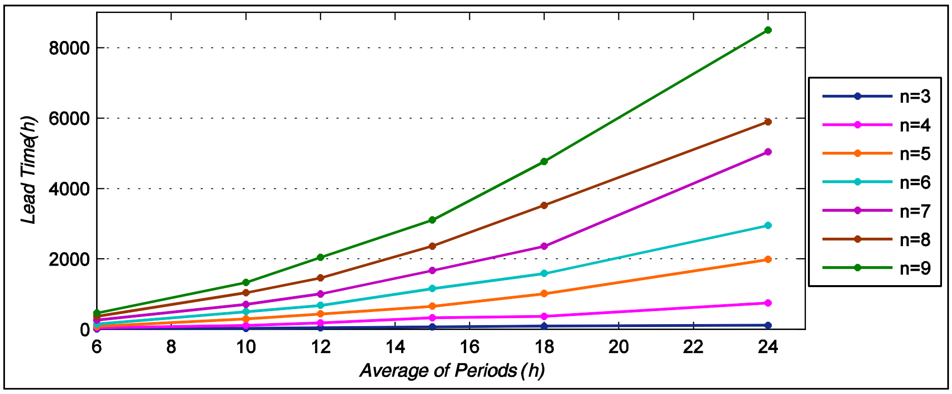

Fig. 6: Relationship of the Stability Time and the Average of Constituent Periods for Constant Number of Constituents values

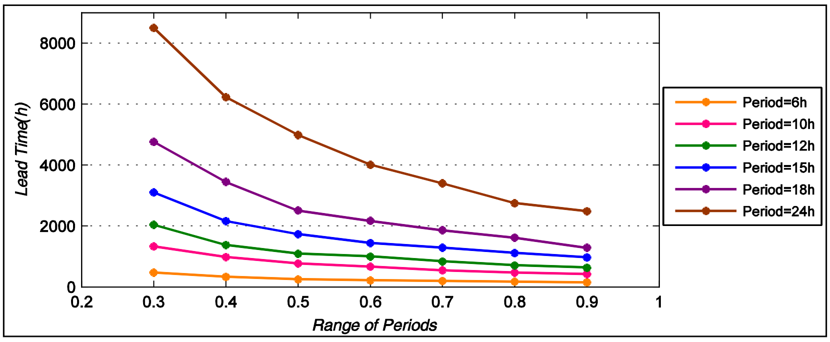

Fig. 7: Relationship of the lead Time and the Time Span for Constant Average of Period values

Evidently, the function of the lead time versus the period span is a power function as well adhering to $T = \alpha l^{-1}$ with acceptable accuratcy.

Hence, the relationship between the lead time of two constituent groups with equal number of group members and period average and different period spans was derived as follows in equation (14):

$$\lambda = \frac{T_{l1}}{T_{l2}} = \left(\frac{l_2}{l_1}\right) \tag{14}$$

where λ is the period span coefficient, T_{l1} is the lead time of the constituents group with period span of l_1 and T_{l2} is the stability time of the constituent group with period span of l_2. Therefore, the effects of interplaying factors have been investigated on the lead time. Hence, the lead time of a real constituents group may be estimated by incorporating all influencing factors in the relationship as proposed in equation (15):

$$T_t = 1.2\ \eta.\mu.\lambda.\ T_0 \tag{15}$$

where T_t is the lead time for any real tidal constituents group, T_0 the lead time of that definite pattern which has been derived using the relative coefficient and the standard deviation of real constituents group. Moreover, as described earlier, η, μ and λ are the number coefficient, average of periods and period the length of the range of periods, respectively. These coefficients should be calculated based on the used so-called, "exhibited pattern". For example, the aforementioned pattern is nine imaginary constituents' spectrum (Fig. 4), with the constant scalar value of 1.2 being the empirical certainty coefficient to assure the sufficiency of the length of the data record.

3. Results and Discussion

3.3. Selection of Tidal Constituents according to Energy Criterion

This study demonstrated the relative merits of using an Energy criterion over the Rayleigh criterion in noise-free circumstances. Therefore, the least squares method of tidal analysis has to be compatible with the energy criterion.

In the proposed new approach, the sequence of tidal constituent selection for the analysis remains in agreement with the corrected tidal potential amplitude method of Cartwright and Edden (1973) however, the inclusion or omission of a certain constituent is governed by equation (15).

The constituents within the same neighboring group such as diurnal or semidiurnal kinds are sorted according to the tidal potential amplitude values. The first three constituents are selected and their respective lead time is investigated via employing equation (15). If the time span of the prepared record is greater than the lead time, the fourth constituents will be added on and the process will be continued. (Table 5).

For instance, assuming that adding the 7th constituent would result in the group stability time for these 7 constituents becoming greater than the prepared data record length, then the 7th constituent shall be omitted, while the 8th one is added and the process is continued.

Table 5. Sorted Semidiurnal Tidal Constituents According to decreasing Tidal Potential Amplitudes

No	Cons name	Period (h)	Tid poten amp* 10^5	No	Cons name	Period (h)	Tid poten amp* 10^5
1	M2	12.4206	90809	10	EPS2	13.12727	671
2	S2	12	42248	11	LDA2	12.22177	670
3	N2	12.65835	17386	12	ETA2	11.75452	643
4	K2	11.96724	11498	13	R2	11.9836	355
5	NU2	12.626	3302	14	H1	12.43822	313
6	MU2	12.87176	2776	15	H2	12.40303	277
7	L2	12.19162	2597	16	OQ2	13.16223	259
8	T2	12.01645	2476	17	MKS2	12.3855	240
9	2N2	12.90537	2301	18	MSN2	11.78613	230

3.2. Assessment of the Stability Time for the Main Tidal Constituents

Thus far, using the Rayleigh criterion, the lead time has been specified to be more than that required to separate tidal constituents. For example, using the Rayleigh criterion, 369 days of hourly data is needed to extract 20 to 30 constituents with adequate separation of closely spaced constituents in the least squares method (Chang and Lin, 2006). In this paper, the lead time was investigated for 68 tidal constituents with the outcome as presented in Table 6:

These constituents have been divided into four groups: six constituents in the slower than diurnal group, twenty one in the diurnal group, eighteen in the semidiurnal group and twenty three in fourth group including terdiurnal, shallow water and other constituents. For comparison purposes, the lead time parameter was denoted *8,767* hours by Foreman in 1977 using Rayleigh criterion.

Employing the Energy criterion, the lead time was calculated separately for each group. The estimated lead times for each of the four groups are as shown in Table 6. Therefore, it may be inferred that the lead time for the system combined of these 68 constituents is to be controlled by the diurnal group which amounts to *3,241* hours or 135 days of hourly data.

In order to illustrate the guidelines for the proposed alternative approach based on the Energy criterion, another hypothetical analysis is explained hereforth: For each constituent, an amplitude equal to its tidal potential amplitude divided by 0.4, is considered as a reasonable value and an imaginary phase angle are specified. Using the software to solve the aforementioned system of equations, tides are predicted for longer duration while solely using 135 days for analysis, respectively (Table 7).

Table 6. Main Tidal Constituents Included in Lead Time Assessment

Group name	Constituents	Lead time (h) from Energy criterion
slower than diurnal	SA, SSA, MSM, MM, MSF, MF	2214
diurnal	ALP1, 2Q1, SIG1, Q1, RHO1, O1, TAU1, BET1, NO1, CHI1, PI1, P1, S1, K1, PSI1, PHI1, THE1, J1, SO1, OO1, UPS1	3241
semidiurnal	OQ2, EPS2, 2N2, MU2, N2, NU2, H1, M2, H2, MKS2, LDA2, L2, T2, S2, R2, K2, MSN2, ETA2	2723
others	MO3, M3, SO3, MK3, SK3, MN4, M4, SN4, MS4, MK4, S4, SK4, 2MK5, 2SK5, 2MN6, M6, 2MS6, 2MK6, 2SM6, MSK6, 3MK7, M8, M10	1468

Table 7. Hypothetical Characteristics & Results of Energy Criterion Approach for duration of 135 Days for 68 Constituents

Cons.*	**Tidal Potential Amp.**	**Artificial characteristics**		**Results of analysis for 135 days**		**Cons.**	**Tidal Potential Amp**	**Artificial characteristics**		**Results of analysis for 135 days**	
		Amp.	Phase	Amp.	Phase			Amp.	Phase	Amp.	Phase
SA	0.01156	0.02890	5	*0.02861*	*5.4360*	M2	0.90809	2.27022	175	*2.27129*	*174.99*
SSA	0.07281	0.18202	10	*0.18197*	*9.9643*	H2	0.00277	0.00692	180	*0.00609*	*174.02*
MSM	0.01579	0.03947	15	*0.03947*	*15.007*	MKS2	0.0024	0.006	185	*0.00600*	*188.99*
MM	0.08254	0.20635	20	*0.20635*	*19.998*	LDA2	0.0067	0.01675	190	*0.01674*	*189.99*
MSF	0.01369	0.03422	25	*0.03423*	*24.988*	L2	0.02597	0.06492	195	*0.06491*	*195.00*
MF	0.15647	0.39117	30	*0.39118*	*29.999*	T2	0.02476	0.0619	200	*0.06178*	*200.00*
ALP1	0.00278	0.00695	35	*0.00696*	*34.903*	S2	0.42248	1.0562	205	*1.05640*	*205.01*
2Q1	0.00955	0.02387	40	*0.02386*	*40.038*	R2	0.00355	0.00887	210	*0.00892*	*207.97*
SIG1	0.01152	0.02880	45	*0.02885*	*44.937*	K2	0.11498	0.28745	215	*0.28733*	*215.01*
Q1	0.07217	0.18042	50	*0.18046*	*49.987*	MSN2	0.0023	0.00575	220	*0.00575*	*219.94*
RHO1	0.01371	0.03427	55	*0.03437*	*54.999*	ETA2	0.00643	0.01607	225	*0.01607*	*224.99*
O1	0.37694	0.94235	60	*0.94253*	*59.996*	MO3	0.0022	0.0055	230	*0.00550*	*229.99*
TAU1	0.00493	0.01232	65	*0.01255*	*64.588*	M3	0.01188	0.0297	235	*0.02970*	*234.99*
BET1	0.00278	0.00695	70	*0.00711*	*69.562*	SO3	0.00118	0.00295	240	*0.00295*	*239.98*
NO1	0.02964	0.07410	75	*0.07468*	*75.037*	MK3	0.0035	0.00875	245	*0.00875*	*245.00*
CHI1	0.00567	0.01417	80	*0.01449*	*79.345*	SK3	0.0027	0.00675	250	*0.00675*	*250.00*
PI1	0.01028	0.0257	85	*0.01520*	*114.09*	MN4	-	0.003	255	*0.00300*	*255.00*
P1	0.17543	0.43857	90	*0.48378*	*85.604*	M4	-	0.004	260	*0.00400*	*259.99*
S1	0.00416	0.0104	95	*0.09141*	*291.37*	SN4	-	0.005	265	*0.00500*	*265.00*
K1	0.53011	1.32527	100	*1.24577*	*104.07*	MS4	-	0.006	270	*0.00600*	*269.99*
PSI1	0.00422	0.01055	105	*0.05840*	*56.018*	MK4	-	0.007	275	*0.00700*	*275.00*
PHI1	0.00755	0.01887	110	*0.02434*	*118.70*	S4	-	0.008	280	*0.00800*	*280.00*
THE1	0.00567	0.01417	115	*0.01419*	*115.21*	SK4	-	0.009	285	*0.00900*	*285.00*
J1	0.02964	0.0741	120	*0.07404*	*119.83*	2MK5	-	0.01	290	*0.01000*	*290.00*
SO1	0.00215	0.00537	125	*0.00530*	*125.09*	2SK5	-	0.011	295	*0.01100*	*295.00*
OO1	0.01624	0.0406	130	*0.04057*	*130.00*	2MN6	-	0.012	300	*0.01200*	*300.00*
UPS1	0.00311	0.00777	135	*0.00778*	*134.99*	M6	-	0.013	305	*0.01300*	*305.00*
OQ2	0.00259	0.00647	140	*0.00647*	*139.97*	2MS6	-	0.014	310	*0.01400*	*310.00*
EPS2	0.00671	0.01677	145	*0.01676*	*145.01*	2MK6	-	0.015	315	*0.01500*	*315.00*
2N2	0.02301	0.05752	150	*0.05750*	*150.00*	2SM6	-	0.016	320	*0.01600*	*320.00*
MU2	0.02776	0.0694	155	*0.06936*	*154.99*	MSK6	-	0.017	325	*0.01700*	*325.00*
N2	0.17386	0.43465	160	*0.43472*	*160.00*	3MK7	-	0.018	330	*0.01800*	*330.00*
NU2	0.03302	0.08255	165	*0.08261*	*165.00*	M8	-	0.019	335	*0.01900*	*335.00*
H1	0.00313	0.00782	170	*0.00802*	*173.38*	M10	-	0.02	340	*0.02000*	*340.00*

*Cons.: Constituents, See text below for explanation.

In the above Table (7), shaded cells showed a more pronounced difference between the hypothetical characteristics of the 6 diurnal tidal constituents and the results of the proposed analysis for a data length of 135 days. These differences emerged because the pertinent periods are very close and concentrated. Of course, these differences are not very significant in an engineering applications sense, because the tidal potential amplitude coefficients of these 4 constituents are relatively quite small. Certainly, the omission of insignificant constituents such as S1 and PSI1 enhances the accuracy of the results.

Finally, performing a continuous analysis and investigating the allowable error per the Energy criterion approach for these 68 constituents revealed that the exact lead time is equal to 2,886 hours, as a testimony to the computational efficiency of the new method utilizing the Energy criterion, noting at the same time that the estimated lead time was greater than the exact one.

4. Conclusion

In this article, a new approach to tidal analysis particularly suitable to records of limited lengths along with guidelines on implementing tidal constituents within the context of a least squares method is presented with the aid of a number of illustrative case studies. In contrast, it has been demonstrated that the Rayleigh criterion in the context of the least square analysis, which is rather disadvantaged in incorporating relative effects of two neighboring constituents only and in ignoring the combined overall effects of all neighboring constituents may, in some applications, be suitably replaced by a proposed Energy criterion. The stability time of the matrix of equations, a key parameter in the new approach to the least squares analysis increases for cases whenever the periods of tidal constituents somewhat concentrate. Upon the introduction of the energy criterion and a relative concentration coefficient to gauge accuracy, a new approach has been established as an alternative to the Rayleigh criterion, whereby considering the relative concentration, standard deviation, and the overall collective effects of all tidal constituents, the lead time has been investigated for principal groups. The selection of the tidal constituents for the new least squares method, considering the tidal potential amplitude for each particular constituent, the time-span of the data, as well as the behavior of the lead time for different groups of constituents were treated in continuation. Eventually, it was demonstrated that 135 days of hourly data was required for reasonably high accuracy of analysis including 68 of the main constituents. By virtue of this new method, the inclusion of more tidal constituents for a fixed data record length became possible in comparison to the Rayleigh method, which could obviously be advantageous for many sites around the globe where long-term data are not available.

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