Non-propagating Waves and Behavior of Curtainwall-pile Breakwaters

Nejadkazem, Omid*; Mostafa-Gharabaghi, Ahmad Reza

Department of Civil Engineering, Sahand University of Technology, Tabriz, IR Iran

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Abstract

Usually, evanescent modes or non-propagating waves are produced when a propagating incident wave impinges on an interface between two media or materials such as curtainwall-pile breakwater (CPB) at a subcritical angle and decay with distance from interface. To achieve an effective prediction of hydrodynamic performance of CPB and overcome the problem of underestimation of energy loss, the effect of non-propagating waves is considered. A numerical model was developed that can predict the interaction of irregular waves normally incident upon a CPB. The developed numerical model is based on an eigenfunction expansion method. The model utilizes a matching boundary condition nearby the vertical piles which accounts for energy dissipation. The accuracy and validity of the method were verified by comparison with results obtained by other authors. Furthermore, the nature of contraction coefficient is discussed and a new predictive formula is presented and compared with the former one suggested by Mei et al. (1974). The proposed contraction coefficient which comprises the hydrodynamic characteristics of waves enhances efficiency and effectiveness of estimation in comparison with other formulas. Non-propagating waves can affect wave propagation and scattering near permeable barriers. These effects have been analyzed and optimum number of necessary corresponding evanescent modes have been studied. Although, it is found that the first 5 evanescent modes strongly alter the results and stabilize them, greater reliability may be achieved by taking twenty waves into account.

Keywords: Curtainwall-pile breakwaters, Evanescent modes, Eigenfunction expansion, Contraction coefficient

1. Introduction

In the past five decades, theoretical and experimental efforts have been made to estimate the properties of two types of dissipative breakwaters, namely perforated and slotted breakwaters (Lamb, 1932; Wiegel, 1961; Jarlan, 1971; Sollitt and Cross, 1972; Mei et al., 1974; Massel and Mei, 1977; Bennet et al., 1992; Isaacscon et al., 1998; Kakuno and Liu, 1993; Suh et al., 2007; Huang, 2007; Ketabdari and Varjavand, 2008; Liu et al., 2008; Huang and Yuan, 2009; Ji and Suh, 2010; Zhu and Zhu, 2010). These types of breakwaters have two main advantages. The former one is dissipation of incident wave energy and the latter one is reduction of wave reflection and cause the reduction of wave forces acting on structure (Liu et al., 2008). In practice, there are different types of porous structures, such as curtainwall breakwater which is the simplest and consists of a vertical wall extending from the water surface to some distance above the
sea bed or a pile breakwater, which consists of an array of closely spaced vertical piles. CPB was originally proposed by Suh et al. (2006), who developed a mathematical model to predict its hydrodynamic characteristics, using eigenfunction expansion method.

In this study, the method proposed by Mei et al. (1974), and later applied by Suh et al. (2007) was adopted. The fluid region was divided into two domains, front and back of the perforated wall. Within each region, distinct solutions were assumed and they were obtained applying the matching condition at the location of perforated wall. Usually in literature, 50 to 100 components of non-propagating waves are considered without any published study. This paper outlines how non-propagating waves affect the wave scattering of CPB and identifies how many evanescent modes must be considered for reliable results. Recently, Nejadkazem and Gharabaghi (2012) introduced a new empirical formula for contraction coefficient as a function of porosity and wave steepness. The efficiency and effectiveness of this new formula is compared with the formula proposed by Mei et al. (1974), while evanescent modes are considered. The numerical model is based on the eigenfunction expansion method that superposes propagating wave and a series of non-propagating waves. The mathematical model adequately well reproduces available experimental results of wave transmission and reflection coefficients; therefore, it can be used for prediction.

2. Modeling

A wave train of height $H_i$ and angular frequency $\omega$, that propagates in water of constant depth $h$, was passed through a thin CPB as shown in Figure 1. Draft of CPB is noted by $d$, thickness of wall by $b$, and diameter of piles by $D$. A Cartesian coordinate system $(x, z)$ is defined with $x$ measured in the wave propagation direction and $z$ is measured from still water level.

![Fig. 1: Definition Sketch of CPB](image)

The distance between the centers of two neighboring piles is denoted as $2A$ and the minimum width of an opening is $2a$. So the porosity of the lower part of the breakwater at $x=0$ is defined as $r_0 = a/A$. Assuming incompressible fluid and irrotational flow, the fluid motion can be described by a velocity potential $\Phi$, which satisfies the Laplace equation within the fluid region. Consequently, $\Phi$ is subjected to the ordinary boundary conditions, at the seabed (no flow condition), free surface (kinematic and dynamic free surface boundary conditions), and finally lateral boundary conditions (in both time and space). Also, appropriate linearization applied where needed (Dean and Dalrymple, 1991; Isaacson et al., 1998). Thus, velocity potential may be expressed as (Isaacson et al., 1998):

$$\Phi(x, z, t) = \text{Re}[C\phi(x, z)exp(-i\omega t)]$$ \hspace{1cm} (1)

Where

$$C = \left(-\frac{igH_i}{2\omega}\right)\frac{1}{\cosh(kh)}$$ \hspace{1cm} (2)

and $\text{Re[ ]}$ denotes the real part of the argument, $i = \sqrt{-1}$, $t =$ time, $k =$ wave number, and $g =$ gravitational constant.

The spatial variation of the velocity potential, $\phi(x, z)$ should be determined in each region.
Moreover, wave number k must satisfy the dispersion equation as follows:

$$\omega^2 = gk \tanh (kh)$$  \hspace{1cm} (3)

Thickness of the wall is very small compared with the wave length, and as such, the wall has no thickness value mathematically. Then \( \phi_1 \) and \( \phi_2 \) (at upward and downward regions of CPB, respectively) must satisfy the following matching conditions at \( x=0 \):

$$\frac{\partial \phi_1}{\partial x} = \frac{\partial \phi_2}{\partial x} = 0 \quad \text{for} \quad -d \leq z \leq 0, \quad x = 0$$  \hspace{1cm} (4)

$$\frac{\partial \phi_1}{\partial x} = \frac{\partial \phi_2}{\partial x} = iG(\phi_1 - \phi_2) \quad \text{for} \quad -h \leq z \leq -d, \quad x = 0,$$

where the subscripts indicate the regions of the fluid domain. By matching conditions, we declare mathematically that horizontal velocities must be matched at the breakwater location.

The first matching condition describes the mode in which horizontal velocities vanish on both sides of the upper impermeable wall of the breakwater. The second matching condition is for the lower part of the breakwater and describes the mode in which horizontal velocities in both regions are equal at the breakwater and the horizontal velocity at the opening is proportional to the difference of velocity potentials, or pressure difference, across the breakwater. The proportional constant \( G \), often called permeability parameter, generally is a complex number.

Finally, by applying the method adopted by Suh et al. (2007), the velocity potential can be expressed in a series of infinite number of coefficients \( A_m \) that are initially unknown as:

$$\phi_1 = \phi_i - \sum_{m=0}^{\infty} A_m \cos [\mu_m (h + z) \exp(\mu_m x)]$$  \hspace{1cm} (6)

$$\phi_2 = \phi_i + \sum_{m=0}^{\infty} A_m \cos [\mu_m (h + z) \exp(-\mu_m x)]$$  \hspace{1cm} (7)

where \( \phi_i = \cosh[k(h + z)] \exp(ikx) \) is the incident wave potential. The wave numbers \( \mu_m \) (\( m = 0, 1, \ldots \infty \)) are the solution of the dispersion relation, \( \omega^2 = -g \mu_m \tan(\mu_m h) \), which has an infinite discrete set of real roots \( \pm \mu_m \) (\( m \geq 1 \)) for non-propagating waves and a pair of imaginary roots \( \mu_0 = \pm ik \) for propagating waves. By taking \( \mu_0 = -ik \), the propagating waves in Eqs. (6) and (7) will correspond to the reflected and transmitted waves, respectively. In addition, taking the positive roots for \( m \geq 1 \), the non-propagating waves will disappear exponentially with distance from the breakwater.

Velocity potentials (6) and (7) represent the incident wave train combined with a superposition of a propagating mode \( (\mu_0 = -ik, m = 0) \) and a series of evanescent modes \( m \geq 1 \) that decay with distance away from the barrier. They satisfy all relevant boundary conditions e.g., free surface and the bottom boundary condition, except the conditions of pressure continuity along the matching boundary and along the permeable barrier. In order to find the unknown coefficients \( A_m \)'s, the matching conditions at the breakwater were used. The Eqs. (6) and (7) are substituted into Eqs. (4) and (5), respectively. Then each obtained equation is multiplied by \( \cos[\mu_m (h + z)] \), and is integrated with respect to \( z \) over its appropriate range, \( (-d, 0) \) for \( \phi_1 \) and \( (-h, -d) \) for \( \phi_2 \). Finally, they are summed up to obtain a matrix equation for \( A_m \) (Suh et al., 2007)

$$\sum_{n=0}^{\infty} C_{mn} A_m = b_n \quad \text{for} \quad n = 0, 1, 2, \ldots, \infty,$$  \hspace{1cm} (8)

where

$$C_{mn} = \mu_m f_{mn}(-d,0) + (\mu_m - 2iG)f_{mn}(-h,-d),$$  \hspace{1cm} (9)

$$b_n = -\mu_0 f_{0n}(-d,0) + f_{0n}(-h,-d),$$  \hspace{1cm} (10)

$$f_{mn}(p,q) = \int_p^q \cos[\mu_m (h + z)] \cos[\mu_n (h + z)] dz$$

$$= \begin{cases} \frac{1}{2} \left[ \frac{\sin[(\mu_m + \mu_n)(h + z)]}{\mu_m + \mu_n} + \frac{\sin[(\mu_m - \mu_n)(h + z)]}{\mu_m - \mu_n} \right] & \text{for} \quad m \neq n \\ \frac{1}{4\mu_m} \left[ 2\mu_m (h + z) + \sin[2\mu_m (h + z)] \right] & \text{for} \quad m = n \end{cases}$$  \hspace{1cm} (11)
To obtain the numerical solution for the problem, Eq. (8) is truncated into a finite number of terms (N). This leads to a complex matrix equation of rank N which can be solved for the first N unknown coefficients \( A_m \). The mathematical method of solving complex matrix equations can be found in Musto et al. (2001) in more detail. Once these coefficients are calculated, the various quantities of engineering interest may readily be accessed. The (real) transmission and reflection coefficients, denoted by \( C_t \) and \( C_r \), respectively, are defined as the appropriate ratios of wave heights by:

\[
C_t = \frac{H_t}{H_i}, \quad C_r = \frac{H_r}{H_i}
\]

where \( H_i \) and \( H_r \) are the transmitted and reflected wave heights, respectively. Scattering coefficients are given in terms of \( A_m \) by:

\[
C_t = |1 + A_0| \quad \text{ and } \quad C_r = |A_0| \tag{12}
\]

Due to energy conservation, they are related to the energy dissipation coefficient, \( C_e \) by:

\[
C_t^2 + C_r^2 + C_e = 1 \tag{13}
\]

where \( C_e \) is the proportion of the incident wave energy flux that is dissipated by the barrier.

Finally, the runup \( R_u \) on the upwave face of the barrier, the maximum horizontal force per unit width on the barrier, \( F_{\text{max}} \), and the maximum overturning moment per unit width on the barrier about the seabed, \( M_{\text{max}} \), are given in terms of the coefficients \( A_m \) by the following expressions (Isaacson et al., 1998):

\[
R_u = \frac{H_t}{2} \left( 1 - \frac{1}{\cosh(kh)} \sum_{m=0}^{\infty} A_m \cos(\mu_m h) \right) \tag{14}
\]

\[
F_{\text{max}} = \rho g H_i \frac{1}{\cosh(kh)} \left[ \sum_{m=0}^{\infty} A_m \left( \sin(\mu_m h) - r_o \sin(\mu_m (h - d)) \right) \right] \tag{15}
\]

\[
M_{\text{max}} = \rho g H_i \frac{1}{\cosh(kh)} \left[ \sum_{m=0}^{\infty} A_m \left( \cos(\mu_m h) + \mu_m h \sin(\mu_m h) - r_o \cos(\mu_m (h - d)) - r_o \mu_m (h - d) \sin(\mu_m (h - d)) - 1 + r_o \right) \right] \tag{16}
\]

As common in any type of database, to reduce redundancy of predicted hydrodynamic responses, normalization process was adopted. To normalize data, values of Eqs. (16), (17), and (18) were divided by their corresponding scaling values, related to a specific limiting case. Specified limiting case was vertical wall breakwater. Therefore, corresponding scaling values, are predicted values of Eqs. (16), (17), and (18) based upon a zero porosity. These scaling values for force, momentum, and wave run-up are denoted by superscript s as will be used in following Figures.

### 3. Calculation of permeability parameter and contraction coefficient

In this study, the method proposed by Mei et al. (1974) for calculation of permeability parameter was adopted and \( G \) was expressed by:

\[
G = \frac{1}{(\beta/\omega)^{-il}} \tag{19}
\]

where \( \beta \) is energy dissipation coefficient derived by linearizing the nonlinear convective acceleration term in the equation of motion, and \( l \) is the length of jet flowing through the gap between piles. In this equation \((\beta/\omega)\) represents the resistance of breakwater and \( il \) is imaginary part which is associated with the phase difference between the velocity and the pressure due to inertial effects.

The linearized dissipation coefficient \( \beta \) is given by Kim (1998) as follows (Suh et al., 2007)

\[
\beta = \frac{8\alpha}{9\pi} H_i \omega - \frac{1}{\sqrt{(R + 2)^2 + P^2}} \frac{5 + \cosh(2kh)}{2kh + \sinh(2kh)} \tag{20}
\]

where \( P = lk, R = \beta k/\omega \), and \( \alpha \) is the head loss coefficient. Inertial effects and resistance of breakwater can be represented and formulated alternatively (e.g. Kakuno and Liu, 1993; Park et al., 2000, among others).

The jet length which is related to blockage coefficient \( C \) is shown by Suh et al. (2002) as:

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\]

\[
M_{\text{max}} = \rho g H_i \frac{1}{\cosh(kh)} \left[ \sum_{m=0}^{\infty} A_m \left( \cos(\mu_m h) + \mu_m h \sin(\mu_m h) - r_o \cos(\mu_m (h - d)) - r_o \mu_m (h - d) \sin(\mu_m (h - d)) - 1 + r_o \right) \right] \tag{18}
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\]
Kakuno and Liu (1993) presented the blockage coefficient for circular piles by:

$$C = \frac{\pi}{4} A (1 - r_0)^2 \frac{1}{1 - \xi}, \quad \xi = \frac{\pi^2}{12} (1 - r_0)^2$$

(22)

This formula is an approximate relation and its accuracy is enhanced when porosity increases. Park et al. (2000) proposed the following formula for head loss coefficient:

$$\alpha = \left( \frac{1}{r C_r} - 1 \right)^2$$

(23)

where ad hoc porosity $r$ is given by:

$$\frac{1}{r^2} = \frac{1}{D} \int_{-b/2}^{b/2} dx \frac{r(x)^2}{D}$$

(24)

by defining spatially varying porosity as:

$$r(x) = 1 - \frac{\sqrt{(D/2)^2 - x^2}}{(D/2) + a}$$

(25)

and $C_r$ is the contraction coefficient for which the following empirical relation is suggested by Mei et al. (1974)

$$C_r = 0.6 + 0.4 r_0^2$$

(26)

It is well known that eq. (26) is an empirical formula adopted from steady flow passed a sharp-edged orifice but actually, the contraction coefficient mainly depended on orifice geometry and for thick or rounded edges; its value should get closer to unity. Moreover, it is expected that hydrodynamic characteristics of flow must affect this coefficient. Therefore, in this paper, the new formula proposed by Nejadkazem and Gharabaghi (2012) for estimation of contraction coefficient is used and compared with previous one. This formula includes the significant wave characteristics in addition to the geometry of breakwater and will be discussed in the next section.

The proposed contraction coefficient not only represents vena contracta effect of slotted nature of barriers but also, can implement resistances produced by barrier which is not included explicitly in the linearized dissipation coefficient. The aforementioned resistances are referred as major loss in literature and dominantly caused by viscous effects. The non-propagating waves that are produced in the vicinity of barrier are formed in the shape of disturbance in flow pattern (Tezaur et al., 2008). Therefore, they may be considered responsible for these resistances related to presence of breakwater in wave propagation region. The effect of these non-propagating waves can be modeled directly in the permeability parameter and it is rational to be included in the energy dissipation coefficient due to its nature and origin of initiation. The Therefore, proposed contraction coefficient contains non-propagating wave effects (in the form of disturbance). Consequently, in comparison with the numbers proposed by previous researchers (e.g. Rageh and Koarim, 2010; Nejadkazem and Gharabaghi, 2011 among others); Fewer number of evanescent modes should be considered in modeling to be confident about accuracy of results. Non-uniform distribution of non-propagating waves (in front of barrier and in its leeside) is one of the reasons that the so called, “contraction coefficient” is not the same for wave reflection and transmission, so different formulas were proposed for each one as follows:

$$C^c_R = 1 - 1.60 \times (1 - r_0) + 1.13 \times (1 - r_0)^{2.54} \times \exp \left( 7.12 \times \frac{H_s}{L_s} \right)$$

(27)

$$C^c_T = 1 + 0.47 \times (1 - r_0) - 1.09 \times (1 - r_0)^{0.4} \times \exp \left( -4.59 \times \frac{H_s}{L_s} \right)$$

(28)

The mean value of these formulas is recommended for computing wave run-up, wave forces, and overturning moments.

4. Extension to irregular waves

For estimation of the reflection and transmission coefficients of irregular waves, the energies of incident, reflected and transmitted waves, $E_i$, $E_r$, and $E_T$ within the range of $f_{min}$ to $f_{max}$ (effective frequency range of resolution) must be calculated. This is accomplished by:
\[ E_i = \int_{f_{\min}}^{f_{\max}} S_i(f) df \quad E_r = \int_{f_{\min}}^{f_{\max}} S_r(f) df \]

\[ E_i = \int_{f_{\min}}^{f_{\max}} S_i(f) df \quad E_c = \int_{f_{\min}}^{f_{\max}} S_c(f) df \]  \hspace{1cm} (29)

where \( S_i, S_r, S_c \) and \( S_c \) are incident, reflected, transmitted, and dissipated wave energy density spectra. Reflection and transmission coefficients for each frequency component based upon aforementioned regular wave model, are calculated and denoted by \( K_r(f) \) and \( K_c(f) \). Goda spectrum was applied for incident wave spectrum. For the computation of energy dissipation coefficient, root-mean-squared (RMS) wave height was used instead of the incident wave height \( H_i \). Therefore, wave scattering and energy dissipation of random waves could be calculated by superposition of component waves. The RMS wave height was calculated by:

\[ H_{rms} = H_s / \sqrt{2} \]

For any particular frequency component, different spectral densities were calculated by:

\[ S_i(f) = \left| K_r(f) \right|^2 S_i(f) \quad S_r(f) = \left| K_c(f) \right|^2 S_r(f) \]  \hspace{1cm} (30)

\[ S_c(f) = \left( 1 - \left| K_r(f) \right|^2 - \left| K_c(f) \right|^2 \right) S_c(f) \]  \hspace{1cm} (31)

The energies of the incident, reflected and transmitted waves were proportional to the squares of the respective wave heights. The reflection, transmission and dissipation coefficients can be defined as follows:

\[ C_r = \sqrt{E_r / E_i} \quad C_t = \sqrt{E_t / E_i} \quad C_c = E_c / E_i \]  \hspace{1cm} (32)

The irregular wave spectra of wave run-up, maximum horizontal force and maximum overturning moment are calculated by:

\[ S_R(f) = \left[ T_R(f) \right]^2 S_i(f) \quad S_F(f) = \left[ T_F(f) \right]^2 S_i(f) \]

\[ S_M(f) = \left[ T_M(f) \right]^2 S_i(f) \]  \hspace{1cm} (33)

where \( T_{R,F,M}(f) \) are frequency-dependent transfer functions of wave run-up, maximum horizontal force, and maximum overturning moment, respectively. They can be computed by using equations of (16)-(18) as:

\[ T_R = \frac{R_F}{H_i} \quad T_F = \frac{F_{max}}{H_i} \quad T_M = \frac{M_{max}}{H_i} \]  \hspace{1cm} (34)

The zeroth moments of wave run-up, maximum horizontal force, and maximum overturning moment can be determined by:

\[ R_{mo} = \int_{f_{\min}}^{f_{\max}} S_R(f) df \quad F_{mo} = \int_{f_{\min}}^{f_{\max}} S_F(f) df \]

\[ M_{mo} = \int_{f_{\min}}^{f_{\max}} S_M(f) df \]  \hspace{1cm} (35)

The dimensionless zero-moment wave run-up, maximum horizontal force, and maximum overturning moment are defined as:

\[ \hat{R}_{mo} = \frac{R_{mo}}{R_{mo}} \quad \hat{F}_{mo} = \frac{F_{mo}}{F_{mo}} \quad \hat{M}_{mo} = \frac{M_{mo}}{M_{mo}} \]  \hspace{1cm} (36)

The superscript \( s \) denotes limiting values of wave run-up, maximum horizontal force, and maximum overturning moment, when porosity approaches to zero. These limiting cases for maximum horizontal force, maximum overturning momentum, and wave run-up are also denoted by superscript \( s \) in Fig. 4.

5. Verification of numerical results by experimental measurements

In this section, the results of mathematical model are compared with experimental results of Suh et al., (2007). All experiments were conducted at a water depth of 0.37m. Circular piles of 7cm diameter were used with \( a = 1.5, 2.33, 3.5, \) and 5.25cm, which corresponded to 7,6,5, and 4 piles, respectively, in the flume of 0.7m width. The corresponding porosity of the lower perforated wall was 0.3, 0.4, 0.5, and 0.6, respectively. The thickness of the curtain wall (b) was 3.5cm. Five different drafts of the curtain wall were considered; 12, 14, 16.8, 21.1, and 28.1cm. The curtain wall was high enough above the water level to prevent wave overtopping. For the experimental data, wave height \( H=7.2cm \) were used, which agreed well with the linear wave theory.

In the mathematical model, the number of terms (evanescent modes) used in the eigenfunction expansion method was 50 terms, in order to compare two different formulas of contraction coefficient and validate modeling itself. Figure 2 shows the comparisons between measured and predicted reflection and transmission coefficients for different relative water depth and porosities as a function of relative draft \((d/h)\). In general, the mathematical model adequately reproduces most important features of the experimental results.
Figure 2 shows that the reflection and transmission coefficients, increase and decrease, respectively with the relative water depth in both prediction and measurement. Also, it is noticeable that the effect of relative water depth diminishes at extremes of relative draft.

Figure 3 compares the measurements of Suh et al. (2007) with the predicted reflection and transmission coefficients as a function of porosity for different relative drafts. As it is shown, increasing porosity between piles leads to decrease of reflection coefficient and increase of transmission coefficient. Also increasing the water depth, leads to diminishing the effect of porosity; because wave motion vanishes in the lower part of the water surface for deep water.
It can be elucidated for Figures 2 and 3 that the formula proposed by Nejadkazem and Gharabaghi (2012) may generally reproduce experimental results more accurately in comparison with formula of Mei et al. (1974). The priority of latter formula to former one is enhanced for shallow water, where breakwaters are constructed; therefore, the latter one has been utilized.

6. Results

After verification of the mathematical model; effect of evanescent modes on accuracy and stability of method is shown in Fig. 4. This figure shows the variation of transmission, reflection, and dissipation coefficients against number of evanescent modes, based upon contraction coefficients proposed by Mei et al. (1974) and Nejadkazem and Gharabaghi (2012) for both regular and irregular waves. Variation of wave run-up, maximum horizontal force and overturning moment are also demonstrated in similar way.

It is easily concluded that the first few evanescent modes will strongly alter the numerical results and later, numerical model gains stability. However, in order to be confident about the accuracy of results, in this paper, the first twenty evanescent modes are used in eigenfunction method. Moreover, wave run-up, maximum horizontal wave force, and maximum overturning moment about the mud line are calculated for irregular waves and presented as a function of porosity for five significant wave periods and relative drafts of CPB in Figure 5. The wave periods of 0.61, 0.78, 1.05, 1.88, and 8.17 s corresponds to relative water depth \((kh)\) 4, 2.5, 1.5, 0.7, and 0.15, respectively. Increasing in porosity leads to reduction of wave run-up, maximum horizontal force, and maximum overturning moment about mud line, especially for shallow and intermediate water wave condition (i.e., long waves).
From Figure 5, it could be noticed that, when relative water depth is 4 or 2.5 m, maximum horizontal force, overturning moment and wave run-up is not affected by porosity, while for relative water depth less than 2.5 m, porosity plays an effective role in decreasing amount of aforementioned parameters. For example, maximum horizontal force exerted on an impermeable breakwater, can be reduced by about 59.0% and that of the maximum overturning moment on mud line by about 46% by making breakwater permeable with a porosity of 0.45. In addition, wave run-up can be reduced by about 27.5%; when relative water depth is 1.5 and relative draft is 0.15.

Figure 5 also suggests that for d/h=0.35, same results of former relative draft (i.e. 0.15) could be drawn out. Maximum horizontal force exerted on an impermeable breakwater, can be reduced by about 65.0% and that of the maximum overturning moment on mud line by about 50% by making breakwater permeable with a porosity of 0.45. Also, wave run-up can be reduced by about 21%; when relative water depth is 0.7 and relative draft is 0.35.

Moreover, we may conclude from Figure 5 that CPBs are not an efficient choice for deep water and their performance is effective in intermediate and shallow waters. Relative draft increases wave force, overturning moment and run-up. But, due to results shown in Figure 2, and role of reducing wave transmission in the tranquility of leeside of breakwater; small relative drafts are not proposed. For deep water, where draft of CPB is considerable, porosity does not have considerable effect on the aforementioned hydrodynamic characteristics. This is because water particle displacement and related wave force and moment, reduces with water depth and in effective depth of water particle activity (as a cause of wave propagation) a vertical wall exists in CPBs. It is emphasized that the effect of porosity in the range of high porosities (above 0.6) is ignorable.

As it is expected, the trend of normalized maximum horizontal force and overturning moment are similar. For relative draft, 0.35 in deep water; normalized run-up is greater than 1.0. This is due to inclusion of the evanescent modes.
relative water depth \((kh)\) of 1.5 m, by increasing the relative draft, wave force, wave overturning moment, and wave run-up increases. But, for relative water depth of 0.15 m, relative draft does not have any effect on wave force and overturning moment and trend in relative wave run-up is vice versa. This is because non-propagating waves are formed in the shape of disturbance in permeable interface of two media. Also, comparison between these two figures reveals that wave force, wave overturning moment, and wave run-up decrease as relative water depth increases.

From Figure 6, it can be concluded that for \(kh=0.15\), maximum horizontal force and overturning moment exerted on an impermeable breakwater, can be reduced by about 10-15\%, by making it permeable with a porosity of 0.4. Also, wave run-up decrease in same conditions is not noticeable. Furthermore, it can be deduced that for \(kh=1.5\), maximum horizontal force exerted on an impermeable breakwater, can be reduced by about 22.5-75\% and that of overturning moment on mud line by about 20-75\% by making it permeable with a porosity of 0.4. Moreover, under the same circumstances, wave run-up can be reduced by about 9-27.5\%, depending on relative draft. It is reminded that the effect of relative draft is negligible for relative depth of 0.15 (at most 4\%). Therefore, it can be concluded that for relative water depth below 0.15, CPBs do not present their exclusive advantage with respect to similar breakwaters (such as pile breakwater), so they are not recommended for relative water depth less than 0.15.

Figure 7 shows the variation of hydrodynamic responses of CPBs for different relative draft and porosities as a function of relative water depth for irregular waves. It can be noticed that wave run-up, maximum horizontal force, and overturning moment increase with increasing relative water depth \((kh)\) for intermediate and deep water wave conditions. This can be due to the less contribution of CPBs in wave energy dissipation for deep water conditions.
This trend is different for very shallow water wave condition and hydrodynamic responses of permeable breakwater increases with decreasing relative water depth. The reason may be due to the increasing effect of trapped non-propagating waves. This kind of waves is formed at the interface of two different media of wave propagation and when water depth is not considerable, their effect becomes considerable. Almost most of hydrodynamic responses of CPBs are magnified with relative draft. It can be noticed that the relative draft is more effective when relative water depth is more than 0.75 for porosity of 0.4. Same conclusion may be drawn out for porosity of 0.6 when relative water depth exceeds 0.5m. Also, porosity is more effective when relative water depth is approximately less than 2m. More over the range of \([0.15\ 2.0]\) is recommend for relative water depth, because wave force and run-up are reduced considerably. By considering Figures. 6, and 7 and according to the previous research, it can be concluded that the effective domain of relative draft is \([0.20 - 0.30]\). Since, the CPBs can operate more effectively and
larger relative drafts enhance the wave force, overturning moment and run-up considerably. It should also be noticed that the effect of porosity and relative water depth on wave run-up diminishes when relative water depth exceeds 2.0. In the mentioned domains of relative water depth, not only porosity and relative draft can affect wave force, overturning moment, and runup, but also porosity can reduce wave reflection.

Figure 8 shows the variation of wave dissipation, transmission and reflection with respect to porosity for different relative water depths and drafts. It can be noticed that the dissipation coefficient can reach to 0.5 for porosity of about 0.1 for different relative water depths and drafts. However, increasing relative water depth and draft usually leads to decreasing dissipation coefficient. This result confirms the fact that the performance of CPB is deteriorated in deeper water conditions. Also, result confirms that increasing draft of CPB, increases wave reflection. Furthermore, Figure 8 illustrates that the transmission coefficient decreases with increasing values of both relative water depth and draft. This can be explained by considering the water particle motions. As relative draft increases, the water particles velocity and acceleration through the lower gap increases and enhances turbulence that appears due to this contraction. Consequently, wave energy dissipation is magnified. In addition, as relative draft increases, the area which water pass through decreases and as a result transmitted wave decreases.

Also Figure 8 demonstrates that the reflection coefficient increases with increasing relative water depth and draft. Again, this can be explained by considering water particle motion.

Fig. 8: Wave dissipation, transmission and reflection as a function of porosity for: a, c, i) different relative water depth and $d/h=0.15$; b, f, j) different relative water depth and $d/h=0.35$; c, g, k) different relative draft & $kh=0.15$; d, h, i) different relative draft and $kh=1.5$. 
Finally, it can be elucidated that optimum porosity is located in domain $[0.10-0.3]$. In this domain, maximum energy dissipation is obtained and wave reflection and transmission is maintained in desired range. Furthermore, wave force, overturning moment, and run-up are reduced considerably by making breakwater permeable when porosity is located in optimum domain.

7. Discussion

In this study, the method used by Suh et al. (2007) for studying PCBs is applied. It is based on an eigenfunction expansion method and utilizes a boundary condition at the surface of the permeable barrier that accounts for energy dissipation within the barrier. Expressions are developed for engineering parameters of interest, including the transmission coefficients, reflection coefficients, wave run-up on the upwave face of the barrier, maximum horizontal force, and overturning moment at mud line for both regular and irregular waves. The model was verified with experimental results of Suh et al. (2007), in which comparison was made for various values of relative draft, spacing between piles, and relative water depth.

In general, numerical model is able to adequately reproduce the most important features of the experimental results. However, the reflection coefficients were over-predicted and transmission coefficient were under-predicted. It is concluded that the first few evanescent modes will strongly alter the numerical results and later, numerical model gains stability, when twenty of them are taken into account. CPBs are not an efficient choice for deep water wave and as such, are recommended for intermediate water wave, especially the range of $[0.15-2.0]$, because wave force and run-up are reduced considerably. Also, two main advantages of permeable breakwaters are examined theoretically for irregular waves. The primary one is reduction of wave reflection, transmission (by energy dissipation through resistance, inertial effects of breakwater, and presence of nonpropagating waves) and the secondary one is reduction of wave force, overturning moment and wave run-up.

As the relative water depth increases, the wave run-up, maximum wave force and overturning moment increases as well. On the other hand, decreasing in relative draft, leads to reduction of maximum horizontal wave force, overturning moment and, wave run-up. It can be concluded that CPBs can operate both effectively and efficiently in the range of $[0.2-0.3]$ for relative draft of. It can be noticed that relative draft is more effective for shallow and intermediate water wave condition, but not for deep water. This is more sensible by increasing relative draft. It is found that as the porosity between piles increases, wave run-up, maximum horizontal wave force and overturning moment decreases. Furthermore, it is elucidated that optimum porosity to gain maximum energy dissipation and maintaining wave reflection and transmission in desired range can be located in domain $[0.10-0.3]$. In this domain, wave force, overturning moment, and run-up are reduced considerably by making breakwater permeable. To sum up, recent study suggests that CPBs can be efficient candidate for protecting coastal structures against wave attacks.

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