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Comparative Study of Random Matrices Capability in Uncertainty Detection of Pier's Dynamics

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Abstract

Because of random nature of many dependent variables in coastal engineering, treatment of effective parameters is generally associated with uncertainty. Numerical models are often used for dynamic analysis of complex structures, including mechanical systems. Furthermore, deterministic models are not sufficient for exact anticipation of structure's dynamic response, but probabilistic models are far more efficient in modelling uncertainties. To determine the uncertainty effect on dynamic characteristics, the laboratory samples of a pier structure were created. Stochastic finite element method was used for numerical study of pier's dynamic behavior. For this purpose, the concept of variable distributions of Gaussian and Wishart random matrices for modeling dynamic characteristics of the system was used. At last, by comparing the output frequency response function obtained from empirical experiments and numerical simulation of stochastic finite element method, it is shown that probability density function of Wishart matrix evaluates and tracks uncertainty parameters in system's dynamic response far better than Gaussian matrix.

Keywords:. *Pier, Uncertainty, Stochastic finite element, Frequency response function, Wishart matrix*

1. Introduction

In recent decades, Iran's growing demand for maritime transportation system has emphasized the importance of building costal structures, such as piers in northern and southern coasts of the country. Piers are used for different purposes, such as berthing and mooring, creating facilities for extending tourism and other special purposes (UFC, 2005).

Coastal structures are often damaged severely when huge storms land onshore. Maintaining safe operation of coastal structures throughout its lifetime is the primary principal often considered. Damage to piers during coastal storms around the world, clearly indicates the importance of determining dynamic-treatment changes in these kinds of structures (Clark et al., 2006). This problem becomes more complicated because of uncertainties caused by both random nature of the hydrodynamic loads and administrative problems (Hall et al., 1998). As for random nature of many dependent variables in coastal engineering, treatment of effective parameters have generally uncertainty. Piers are constantly used throughout their lifetime regardless of effects of sea's random waves, various loadings and changing environmental factors. However, majority of loadings and environmental factors can be individually subjected to uncertainty. Uncertainties, defined as relative changes in parameters or extent of error in expressing models, can affect dynamics of coastal

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structures in major ways. To express uncertainty, there are no specific quantities but a range of values to be considered. Inadequate knowledge of uncertainty parameters can lead to an under design and cause damage or to an over design and cost extra.

Numerical models are often used for dynamic analysis of the most complex structures and their mechanical systems. However, probabilistic models are far more efficient in modeling uncertainties and are widely used in computational mechanics over deterministic models which are not sufficient in determining dynamic response of structure as accurately as anticipated. In linear analysis of complex dynamic systems, such as coastal and offshore structures, the uncertainty of the model is treated as subsystems in which their attachment to the main structure is random. Numerous examples of such subsystems are available (for example, loads, pipes, control cables, mobile cranes, waves and so on). Uncertainties are not usually considered in common numerical modelling and precise determination of probable fluctuations are impossible (Soize, 1993; Strasberg and Feit, 1995; Russell and Sparrow, 1995 and Weaver, 1997).

The systems uncertainty theories dealing with probable nature have developed considerably in the past three decades availing different methods for accurate analysing and investigating of uncertainty problems.

Mojtahedi et al (2013) used fuzzy logic systems to investigate about the effect of uncertainty parameters on processing damage detection defects of offshore structures. Using modal analysis for jacket structures, they found out that considering uncertainty in empirical studies and numerical modelling, increased the success rate of the algorithms in studying dynamic behavior of such structures. The roots of the uncertainty in engineering problems generally refer to uncertainties in modelling, physical and operational factors. In general, in these methods, random status of the parameters is modeled by a distribution which their scatter would be around the values of the deterministic system parameters (Schuttrumpf et al., 2008). Therefore, it is better to use statistical methods to analyze and study parametric uncertainty in the system response. If considerable statistical data are available, it would be possible to express the scatter of the model parameters using probabilistic methods. Stochastic finite element method is a new computational method for analyzing the structural uncertainties. In this method, initially, probability density functions (pdf) are fitted upon data, and consequently the model parameters can be indicated as random variables (Adhikari and Manohar, 1999; Adhikari and Manohar, 2000; Sarkar and Ghanem, 2002; Nair and Keane, 2002; Ghanem and Sarkar, 2003; Elishakoff and Ren, 2003 and Sachdeva et al., 2006).

In random fields, constitutive and geometric characteristics can be modeled according to the methods extended to include *the Karhunen-Loeve* method used by Ghanem and Spanos (2003). Such methods are applicable by inversion process of a random system matrix. Their study was on a method based on orthogonal series expansion in the random response field so that they solved the problem using a set of algebraic equations (Ghanem and Spanos, 2003).

Mac and Shorter (2000) suggested a modal method for subsystems properties of which presented a probabilistic distribution, leading to representation of uncertainty of the whole system's modal properties in an equation. In another research, Adhikari and Sarkar (2009) studied the capabilities of Wishart random matrix in determining the uncertainty problem and in modeling the physical uncertainty of structural parameters using a mass-spring oscillator system, randomly attached to a cantilever plate structure.

For dynamic analysis in regions of low frequency, subsystem modeling usually becomes possible via applying a mass system. This strategy leads to preventing addition to the total degrees of freedom. If subsystem modeling in regions of medium frequency is performed through those for low frequency, considerable differences would be observed between calculations and experiments, so it is necessary to consider the internal degrees of freedom of subsystems (Soize, 2000). However, precise information about such subsystem parameters is not available. Therefore, modeling such systems through usual finite element method would be complex, because of a lack of information about geometrical and constitutive properties, geometrical scales, coupling characteristics and so on (Zienkiewicz and Taylor, 1991). To create an appropriate mechanism of transferring energy between primary and secondary structures in higher frequency regions, subsystems should be modeled through sprung-mass systems. If secondary systems are modeled explicitly, this will lead to exact estimation of frequency response function's amplitude and sub-systematic effects on master structure dynamic (Soize, 2000; Strasberg and Feit, 1995).

Dynamic behavior and responses of coastal structures are related to their dynamic characteristics. In modal test, there are different methods such as time and frequency domains to extract modal data from dynamic response of structures. Comprehending changes in dynamic features generally needs a long-term review of a primary structure obtainable from simulated finite element model. However, the structural numerical model, which is provided based on a completely ideal method, cannot express all physical aspects of a real model. Hence, usually in performing modal tests, the obtained frequency response function does not exactly comply with the results of analytic model because of various factors of uncertainty mentioned earlier. In the past decade, a number of studies have focused on the processes of data analysis in modal uncertainty, determination of parameters and methods of reduced data (Soize, 2000).

Despite obvious importance of the uncertainty effect on Pier's behaviors, little is known about the uncertainty effect on Pier's dynamic behavior (Verdure et al., 2005; Schuttrumpf et al., 2008) and it empirically remains almost completely unexplored. Thus, this study was undertaken to evaluate effects of the dynamic uncertainty conditions. For this purpose, a Pier structure was considered based on common dynamic characteristics of plans used to build such structures followed by performing vibration tests on a physical model. Then, ANSYS software was used to model sample of primary structure. Finally, obtained finite element model was used to compare the output responses to experimental results.

In this research, the effect of uncertainties of modeling in laboratory model was applied by a set of randomly distributed sprung-mass. Therefore, based on random matrix theorem a probability model was obtained to express dynamic uncertainties. Random matrices were used to investigate uncertainty problems. To determine random matrices, results obtained from two kinds of Wishart and Gaussian random distributions were compared.

1.1. Wishart and Gaussian Random Probability Matrix Distributions

In practical issues, in which the conventional parametric methods are not efficient, more reasonable solutions based on concept of uncertainty problem can be substituted. For example, to investigate the uncertainty problem if statistical data are few, more random parameters need to be studied.

Regarding this problem, method of using the concept of matrix variate distribution for modeling dynamic system matrices is presented. Hence, the concept of the matrix variate probability density function (pdf) is used in defining a random matrix. The pdf of a random matrix can be defined as random vector. If A is an n×m real random matrix, then the matrix variate pdf of $A \in^{n\times p}$ will be shown PA(A), in which n×m real matrix space is mapped to real line, i.e., $P_A(A):^{n\times m}$

In this study, the pdfs of Wishart and Gaussian random variate matrices relevant to stochastic mechanics are used (Tulino and Verdu, 2004). These pdf of random matrices are defined as follows:

1.2. Wishart Random Matrix

X is an $n{\times}n$ symmetric positive-definite random matrix and $\sum$ is an $n{\times}m$ positive-definite matrix. If $p{\geq}n$, then X has Wishart distribution with the degree of freedom p, and its pdf is as follows:

$$(1)S_X(\mathrm{X}) = \frac{1}{2^{\frac{np}{2}}|\Sigma|^{\frac{n}{2}}\Gamma_p(\frac{n}{2})}|\mathrm{X}|^{\frac{n-p-1}{2}}e^{-\frac{1}{2}tr(\Sigma^{-1}\mathrm{X})}$$

In which $\Gamma_p(\frac{n}{2})$ is the Gamma multi-variate equation and is given by.

$$\Gamma_p(\frac{n}{2}) = n^{p(p-1)/4}\Pi_{j=1}^{p}\Gamma\left[n/2+(1-j)/2\right]$$

In fact, the above equation is generalized for each real amount n>p-1.

1.3. Gaussian random matrix

The random matrix $\mathbf{X} \in \mathbb{R}^{n\times p}$ has Gaussian distribution with the mean matrix $\mu \in \mathbb{R}^{n\times p}$ and the covariance matrix $\mathbf{U}\otimes\mathbf{V}$, where $\mathrm{U} \in \mathbb{R}_+^n$ and $\mathrm{V} \in \mathbb{R}_+^p$ provided the pdf of X is given by:

$$\mathbf{p_x(x)} = \frac{1}{(2n)^{\frac{np}{2}}|\mathbf{U}|^{\frac{p}{2}}|\mathbf{V}|^{\frac{n}{2}}}\mathbf{etr}\left\{-\frac{1}{2}\frac{(\mathbf{x}-\mu)(\mathbf{x}-\mu)^{\mathrm{T}}}{\mathbf{UV}}\right\}$$

This distribution is shown as $\mathrm{X} \sim N_{n,p}(\mu, \mathrm{U}\otimes\mathrm{V})$

In symmetric form, $\mathbf{Y} \in \mathbb{R}^{n\times n}$ is Gaussian random matrix and μ, $\mathbf{U}$ and $\mathbf{V}$ are $n\times n$ matrices, in which commutative relation of $\mathbf{UV}=\mathbf{VU}$ holds. It is said $\mathbf{Y}$ has a symmetric matrix variate Gaussian distribution with mean μ and covariance matrix $\mathbf{B_n^T}(\mathbf{U}\otimes\mathbf{V})\mathbf{B_n}$ and its pdf is given by:

$$p_{\mathbf{Y}}(\mathbf{Y}) = (2\pi)^{-n(n+1)/4}\left|\mathbf{B}_n^{\mathrm{T}}(\mathbf{U}\otimes\mathbf{V})\mathbf{B}_n\right|^{-1/2}\mathbf{etr}\left\{-\frac{1}{2}\mathbf{U}^{-1}(\mathbf{Y}-\mu)\mathbf{V}^{-1}(\mathbf{Y}-\mu)^{\mathrm{T}}\right\}$$

This distribution is shown as

$\mathbf{Y}=\mathbf{Y^T} \sim \mathbf{SN_{n,n}}(\mu, \mathbf{B_n^T}(\mathbf{U}\otimes\mathbf{V})\mathbf{B_n})$ .In which

$\mathbf{U} = E[(\mathbf{X}-\mathbf{M})(\mathbf{X}-\mathbf{M})^{\mathrm{T}}]$

and

$\mathbf{V} = E[(\mathbf{X}-\mathbf{M})^{\mathrm{T}}(\mathbf{X}-\mathbf{M})]/c$

Where c is normalization constant, it relates to $\mathbf{U}$.

The elements of the matrix $B_n \in \mathbb{R}^{n^2\times n(n+1)/2}$ are determined as follows:

$$(B_n)_{ij,gh} = \frac{1}{2}(\delta_{ig}\delta_{jh}+\delta_{ih}\delta_{jg}), \quad i\leq n, \quad j\leq n, \quad g\leq h\leq n$$

In which δ_{ij} is the kronecker's delta.

2. Using the Concept of Probability Distributions in Investigation of Structural Dynamics

In the probability theory, random mass, stiffness and damping matrices are created because of uncertainty in the system. The equation of motion for multi-degree of freedom linear dynamical system is given as follows:

$$\mathbf{M}\ddot{\mathrm{X}}(\mathrm{t})+\mathbf{C}\dot{\mathrm{X}}(\mathrm{t})+\mathbf{K}\mathrm{X}(\mathrm{t}) = \mathrm{f}(\mathrm{t})$$

Where, M, C and K stand for the mass, damping and stiffness matrices, respectively. To solve this dynamic problem, finite element methods are used by considering the mentioned parameters random distributions. It is assumed that $\overline{\mathrm{M}}$, $\overline{\mathrm{C}}$ and $\overline{\mathrm{K}}$ are the mass, damping and stiffness matrices corresponding to the primary system, respectively. These parameters are available using the deterministic finite element method. However, parameters with uncertainty in modeling so M, C and K are actually random matrices whenever a probabilistic is adopted. The matrices $\overline{\mathrm{M}}$, $\overline{\mathrm{C}}$ and $\overline{\mathrm{K}}$ are the best known information concerning the system matrices and they can get closer to the mean of underlying random matrix ensemble of M, C and K. The matrix variate distributions of the random system matrices should be:

1- M, C and K are symmetric matrices.

2- M is positive-definite matrix and C and K are negative-definite matrices.

3- Inverse moments of dynamic stiffness matrix is as follows:

$$\mathrm{D}(\omega) = -\omega^2\mathbf{M}+\mathrm{i}\omega\mathbf{C}+\mathbf{K}$$

Should exist $\forall\omega$. That is, if $\mathbf{H}(\omega)$ is the FRF matrix, given by:

$$\mathbf{H}(\omega) = \mathbf{D}^{-1}(\omega) = [-\omega^2\mathbf{M}+\mathrm{i}\omega\mathbf{C}+\mathbf{K}]^{-1}$$

2.1. Estimating parameters of Wishart matrix

For notational convenience, the notation G is used for any one of the system matrices (M, C and K). Using a least square error minimization approach, Adhikari (2007) suggested Wishart distribution G as follows:

$$\mathbf{G} \sim W_n(p,\Sigma)$$

In which

$$p = n + 1 + \theta_G$$

and

$$\Sigma = \bar{\mathbf{G}}/\alpha_G$$

The constants θ_G and α_G are obtained as follows:

$$\theta_G = \frac{1}{\sigma_G^2}\left\{1 + \frac{\{\text{Trace}(\bar{\mathbf{G}})^2\}}{\text{Trace}(\bar{\mathbf{G}}^2)}\right\} - (n+1)$$

and

$$\alpha_G = \sqrt{\theta_G(n+1+\theta_G)}$$

Here α_G is known as the dispersion parameter which characterizes the uncertainty in the random matrix G. The parameter σ_G is defined as follows:

$$\sigma_G^2 = \frac{\text{E}[\|\mathbf{G} - \text{E}[\mathbf{G}]\|_\text{F}^2]}{\|\text{E}[\mathbf{G}]\|_\text{F}^2}$$

The parameter σ_G is defined as the mean-normalized standard deviation of the random matrix G. The first moment (mean) and the elements of the covariance tensor are determined as follows:

$$\text{E}[\mathbf{G}] = p\Sigma = p\bar{\mathbf{G}}/\alpha_G$$

$$\mathbf{cov(G}_{ij}\mathbf{,G}_{kl}\mathbf{)} = \mathbf{p}(\Sigma_{ik}\Sigma_{jl} + \Sigma_{il}\Sigma_{jk}) = \frac{1}{\theta_\mathbf{G}}(\bar{\mathbf{G}}_{ik}\bar{\mathbf{G}}_{jl} + \bar{\mathbf{G}}_{il}\bar{\mathbf{G}}_{jk})$$

In Equation (18), values for $\bar{G}_{ik}$ are calculated by mean matrix. Therefore, θ_G is the only parameter which controls uncertainty of distribution. θ_G must be calculated by equation (14). σ_G parameter has data related to uncertainty of model which must be calculated by physical experiments or computer simulation.

2.2. Estimating Gaussian Matrix Parameters

In this study, the Gaussian parameters in equation (3) are estimated as follows:

If the random matrix $\mathbf{G} = (\mathbf{G}_{ij})$ (j=1,…, p and i=1,…, n), m times is sampled and is indicated as $\mathbf{G}_k = (\mathbf{G}_{ij})_k$ K=1, ….., n, the mean matrix and the covariance matrix are as follows:

$$\hat{\mu} = \frac{1}{m}\sum_{k=1}^{m}\mathbf{G}_k$$

and

$$\hat{\mathbf{U}} = \frac{1}{m}\sum_{k=1}^{m}(\mathbf{G}_k - \hat{\mu})(\mathbf{G}_k - \hat{\mu})^T$$

and

$$\hat{\mathbf{V}} = \frac{1}{mc}\sum_{k=1}^{m}(\mathbf{G}_k - \hat{\mu})^T(\mathbf{G}_k - \hat{\mu})$$

In which $\mathbf{G}_k$ *is the* k^{th} matrix obtained from the sample.

Process used in this study can be summarized in the following steps:

(1) Definite matrix $G \equiv \{\bar{M}, \bar{C}, \bar{K}\}$ is divided by means of standard finite element method and matrix dimension (n) calculation.

(2) Calculating the normalized standard deviation and the dispersion factor related to the system matrices via experimental or numerical modeling.

(3) Calculation of θ_G using equation (14) for $\mathbf{G} = \{\mathbf{M},\mathbf{C},\mathbf{K}\}$

(4) Determination of p and Σ related to the Wishart matrix using equations (12) and (13), and $\hat{\mu}$, $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ related to the Gaussian matrix by equations (19-21)

(5) Creation of Wishart matrices by means of, wishrnd command in MATLAB software

(6) Creation of Gaussian matrices by means of, *normrnd* command MATLAB software

(7) Repeating of previous steps for system matrices and then its dynamic equation solving via the New mark-β method (Weaver, 1987)

(8) Comparing obtained numerical results from Wishart and Gaussian matrix distributions with experimental results

3. Physical Model and Finite Element Method

To consider the onshore dynamics, a physical model is fabricated based on a typical Pier dimensions to carry out the experiments in a vibration laboratory (Fig. 1). The model is considered weighing 39.12 kg of a deck and 9 steel piles, which are rigidly fixed to the deck. The spring stiffness of each oscillator is 5200 (N/m) The physical and geometrical properties of simulated deck are presented in Table 1.

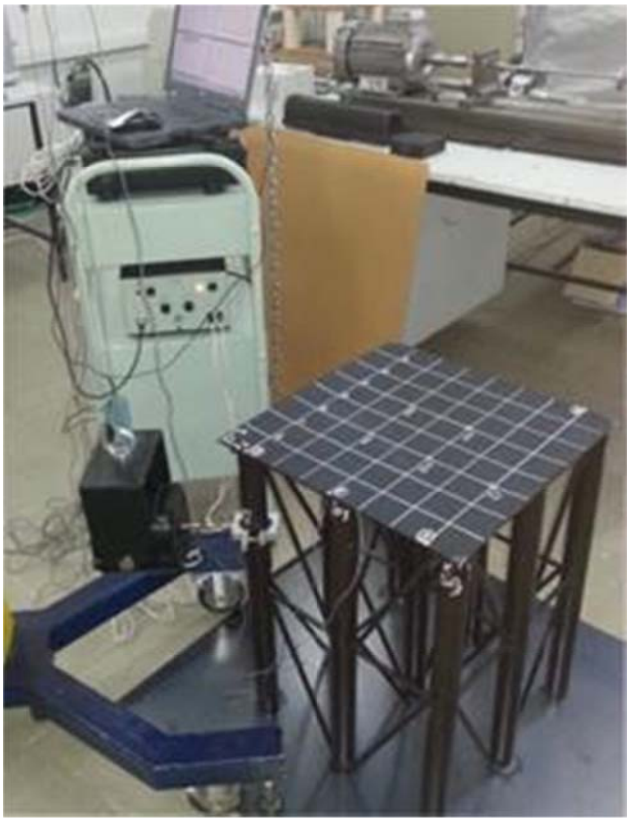


Fig. 1: Configuration of equipment used for experimental model analysis

Table 1: Physical and geometrical properties of the pier deck

Mass density [kg/m^3]	7850
Poisson's ratio	0.3
Young modulus [*GPa*]	200
length [*mm*]	400
width [*mm*]	400
Thickness [*mm*]	10

The piles are steel pipes with exterior diameter, thickness and length of consecutively 40, 5 and 600 mm, in which their geometrical and physical properties are listed in Table 2. As shown in Figure 2, the pile spacing ratio (S/d) is 3.75 (which S is distance and d is diameter of piles). The frame of the model has horizontal and diagonal steel bracing members with 15 and 33.5 cm respectively and 10 mm in diameter.

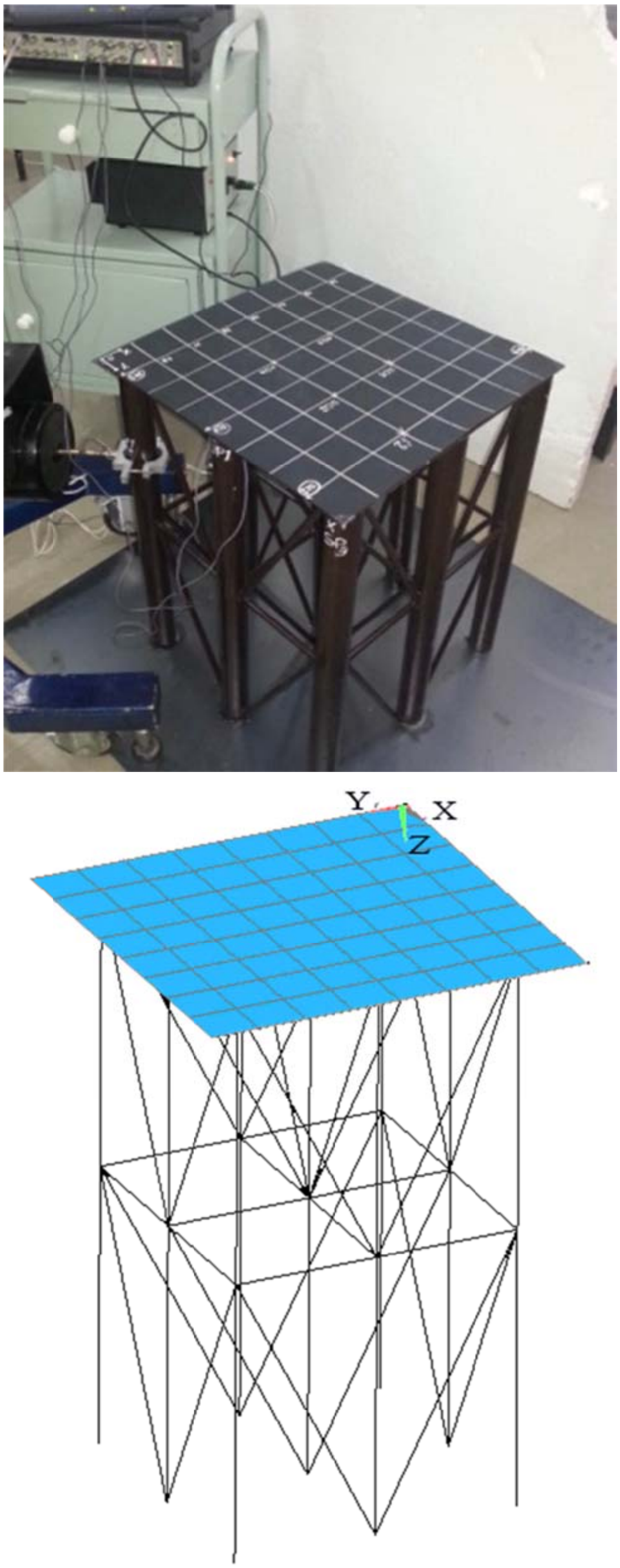


Fig. 2: Mesh related to physical model and numerical model created in ANSYS software

Table 2: Geometrical and physical properties of model pile

Mass density [kg/m^3]	7850
Poisson's ratio	0.3
Young modulus [*GPa*]	200
length [*mm*]	600
Exterior diameter [*mm*]	40
Thickness [*mm*]	5

This pier with homogeneous geometric and uniform Young's modulus and Poisson's ratio defines our baseline system. Using uniform mesh on deck for physical model and numerical model, instead of random sprung-mass distributions, stochastic numerical model is used. The uncertainty appears in the system, because sprung-mass oscillations are attached randomly along the deck of baseline system. The mass of each oscillators attached to the pier's deck is 231.8 g. the deck is divided into 64 elements (8 along the length and 8 along the width) and according to Fig. 2, choosing the corner of the pier deck as the origin, the oscillators and accelerometers are directly attached to nodes. In this study, accelerometers are connected to three points named as SL1, SL4 and SP2 and the model is excited at the point named as f1. Coordinates of these points are seen in Table 3.

Table 3: Coordinates of sensor on the pier

SL1	(0, 5, 0)
SL4	(0, 20, 0)
SP2	(5, 20, 30)
f1	(5, 5, 15)

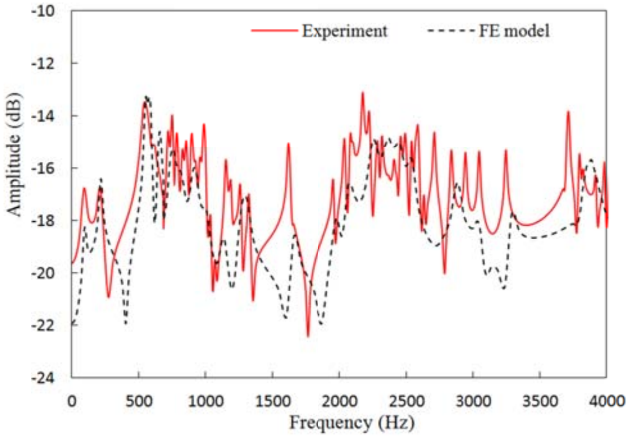
In this study, numerical modeling of pier and its vibration analysis is carried out by ANSYS analytical software pack and MATLAB software is used for developing code for data processing derived from numerical and experimental solution. Excitation is enforced by means of an electro-dynamic exciter (type 4809 B&K), with a force sensor of the type AC20, APTech. The exciter's input is controlled by an amplifier of the type 2706. A lightweight uni-axial accelerometer (type 4508 B&K) is used for recording structural responses. Accelerometer and force sensor is connected simultaneously to a recording system. The frequency sampling of the test setup is selected to be 16.385 kHz and the domain of structural frequency sensitivity based on a primary analysis of finite element model is adapted 0 to 200 Hz. PULSE software was used for further processes of Structural frequency responses.

4. Random Matrix Method in Numerical Analysis

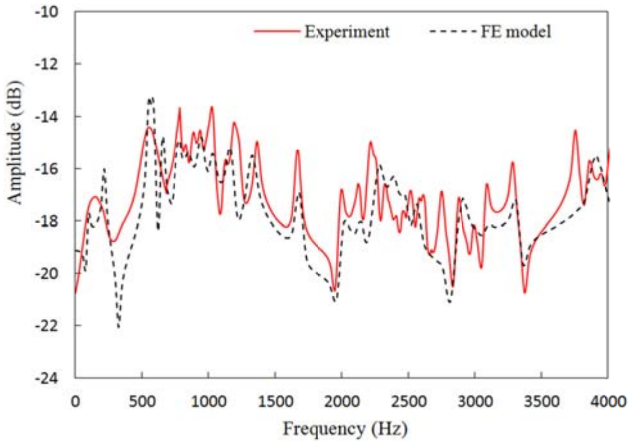
In this section, random matrices related to experimental system which was discussed in section 3 are simulated numerically. The goal is to determine whether the pattern of uncertainty in the experimental data can be described using random matrix. In this study, a pier structure is considered with homogeneous geometry (uniform thickness) and homogeneous structural properties (uniform Young's modulus and Poisson's ratio).

In the experimental analysis, 308 elements are used and finite element model has 1782 degree of freedom and only 120 modes are used in calculation of frequency response function. The damping coefficient is assumed to be 0.7 % for all modes and this constant coefficient for all modes is an assumption. Ideally, damping coefficient should be set for all possible modes and over the 50 times of calculation can be averaged in the experiment. However, considering constant damping coefficient is a usual method used in all similar cases.

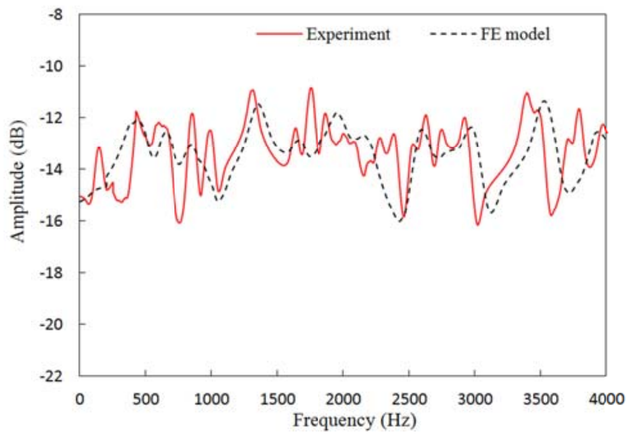
Initially, frequency response function of experimental and finite element model of baseline system in three points are compared in Fig. 3. In this figure, amplitude of Frequency Response Functions (FRFs) of baseline system which are generated by experiment and deterministic finite element model are compared. Generally, both numerical and finite element results are within -16 and -18 db range. The experimental results and finite element model were compared using the criterion of Root Mean Square Error (RMSE). The obtained amount for the points of SL1, SL4 and SP2 were 1.76, 1.20 and 1.26 db, respectively. The main reason for the observed discrepancy between numerical and experimental results might be because of the application of damping coefficient in finite element model.



(a)



(b)



(c)

Fig. 3: Comparison of the amplitude of the FRF for the three points in the baseline system. (a) SL1, (b) SL4 and (c) SP2

Two key parameters are required for employment of random matrix approach, which are mean and normalized standard deviation of system matrices. In Fig. 1, for baseline system, an experimental sample of a pier has been considered. Mean mass and stiffness matrices can be obtained by standard finite element method. All 50 times of the pier and oscillators are separately simulated. Then, samples of stiffness and mass matrices are sorted in matrices with dimension of 1782 by 1782. The normalized standard deviation of the stiffness and mass matrices are obtained using equation (11) as $\sigma_M = 0.00472$ and $\sigma_K = 0.0034$. The value of σ_C equals zero since 0.7% constant modal damping factor is considered for all modes. The values of σ_M and σ_K are only uncertainty associated with the data used in the random matrix method. In fitting Wishart matrices to the obtained data, the information related to the location of attached oscillators and their mass and stiffness properties are not used explicitly. Such model can be proposed to visualize the actual status of a complex engineering system without analyst needing to access comprehensive information about uncertainties. Using n=1782, $\sigma_M = 0.00472$ and $\sigma_K = 0.0034$ and deterministic values of M and K, following values for calculation of the Wishart parameters is achieved:

$$\theta_K = 9675638 \qquad \textit{and} \qquad \theta_M = 98736$$

5. Evaluation of Experimental Results using Random Matrix Theory

In practical engineering, when accessing to statistical information related to uncertainty is not completely plausible, it is desirable to evaluate the efficiency of random matrix fitting. In this section, characteristics of FRFRs are studied in the form of a random function in frequency domain. In the next step, the statistical characteristics of the obtained frequency response functions from experimental tests are compared with those obtained from Wishart and

Gaussian random matrix theories; then statistical characteristics of the amplitude and phase are used for evaluating the predictive capability of random matrix method in relation to the experimental data keeping in mind that the random matrix method only needs limited statistical information to calculate the dispersion parameter (σ_G).

Then, the Monte Carlo simulation using random matrices are compared to empirical results. In Fig. 4, in the state in which excitation is applied at the point of f1, the ensemble mean of FRFs obtained from the experimental data and Monte Carlo simulation at the point of SL1 are compared. According to the present physical model, the measured FRF data is considered up to 4 kHz. In this case, the RMSE empirical results and Wishart and Gaussian numerical results are equal to 1.18 and 1.89 db, respectively. Therefore, as shown in Fig. 4, numerical results obtained from simulation of Wishart random matrices are in correspondent with empirical results. As mentioned earlier, slight difference between results can be due to inaccurate damping coefficient selection. In the ideal case, it is necessary to calculate the amount of modal damping factor accurately in each sample using the experimental results and considering all mode values. However, constant value is used for damping factor by accepting range of the observed differences.

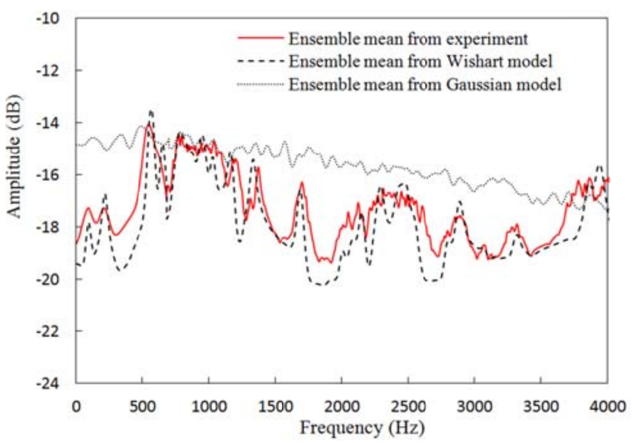


Fig. 4: Comparison of the FRF amplitude of Gaussian and Wishart numerical mean with experimental mean at SL1 point

Similar to steps taken in the previous section, FRFs for points away from the excitation points are shown in Figs. 5 and 6 for SL4 and SP2 points. For these points, the RMSE amounts between empirical mean results and Gaussian and Wishart numerical results are calculated (Table 4). As the results show, numerical results obtained from simulation of Wishart random matrices are more in correspondent with empirical results.

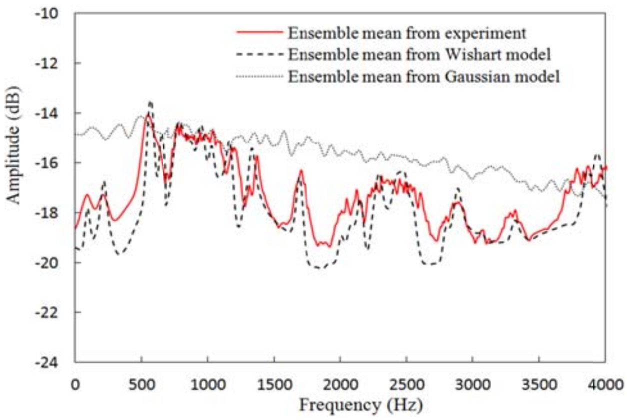


Fig. 5: Comparison of the FRF amplitude of Gaussian and Wishart numerical mean with experimental mean at SL4 point

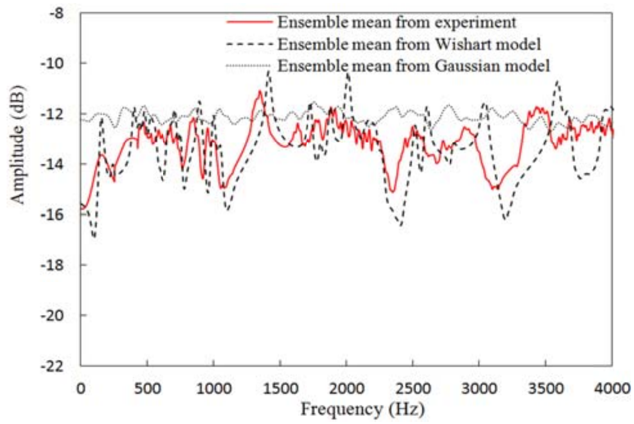


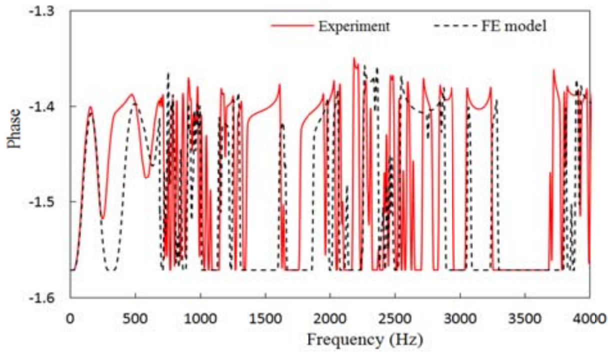
Fig. 6: Comparison of the FRF amplitude of Gaussian and Wishart numerical mean with experimental mean at SP2 point

For the three points mentioned, frequency responses for empirical mean compared with numerical frequency responses includes higher amounts and the pattern of higher points are considerably similar, but their domain amounts are different.

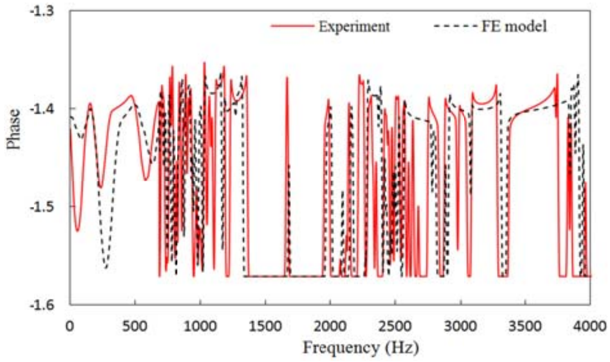
Table 4: Amounts of the square root for errors (dB)

Place of installing sensor	SP2	SL4
Wishart numerical and empirical mean	1.16	0.96
Gaussian numerical and empirical mean	1.4	2.11

In fact, this study clearly shows that selecting constant amounts for modal ratio can lead to considerable errors in anticipation uncertainty of Gaussian and Wishart random matrices. In Fig. 7, the amounts of frequency response function phase obtained from determinate finite element model and empirical results are compared.



(a)



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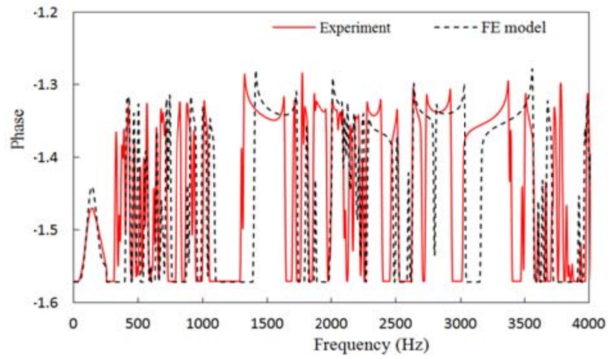

(c)

Fig. 7: Comparison of the phase of the FRF for the three points in the baseline system: (a) SL1, (b) SL4 and (c) SP2

In Fig. 8, cumulative mean of frequency response phase for the point SL1 (close to simulating point) obtained from experiments and simulating Wishart and Gaussian matrices are compared. The pattern of resulting mean for experiment and simulation of Wishart random matrix within frequency domains are similar and for this reason differences between results is possibly caused by false amounts of attenuation ratio.

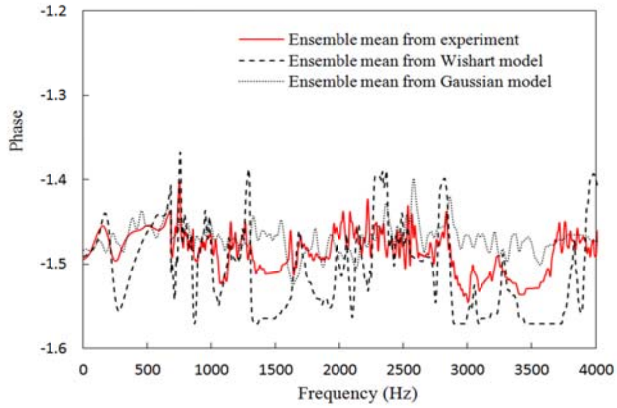

Fig. 8: Comparison of the FRF phase of Gaussian and Wishart numerical mean with experimental mean at SL1 point

Frequency response function for points farther from the stimulated points of SL4, SP2 are consecutively shown in figures 9 and 10. As it is shown in these shapes, empirical mean results in frequency ranges of middle and high are different from simulating random matrix.

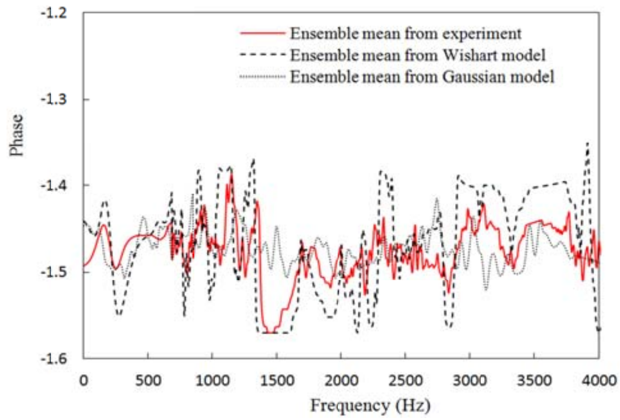

Fig. 9: Comparison of the FRF phase of Gaussian and Wishart numerical mean with experimental mean at SL4 point

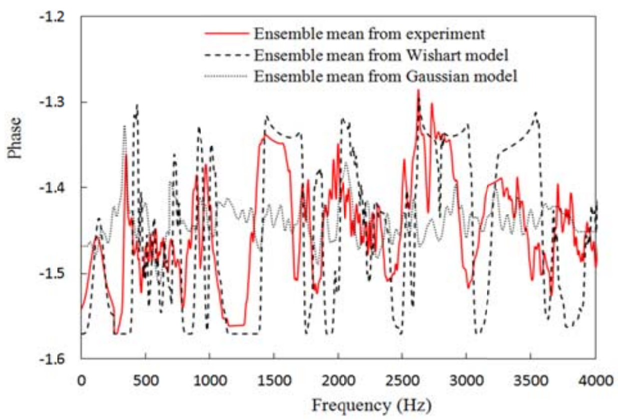

Fig. 10: Comparison of the FRF phase of Gaussian and Wishart numerical mean with experimental mean at SP2 point

Conclusion

Owing to an experimental sample in this research, the effects of the uncertainty parameters in mechanical behavior of coastal pier structure are studied. The existence of uncertainty factors are simulated by means of sprung-mass oscillators which are attached to the deck of pier randomly. Moreover, stochastic finite element method is used in numerical study in which the Wishart and Gaussian random matrix variate distributions for modeling dynamic behavior are applied. Also, the FRF obtained from experimental tests is compared to those with numerical modeling and the following results are obtained:

(1) The probability density functions of both Wishart and Gaussian random variate matrices are compared with experimental results showing that in random mechanical problems Wishart distribution can be a more suitable choice for modeling random system matrices obtained from probability structural mechanics. It is likely due to this fact that Wishart distribution always leads to positive-definite and symmetric matrices.

(2) Constant damping coefficient is chosen which can lead to errors in predicted uncertainty using Gaussian and Wishart random matrices. Furthermore, the composition of the environmental noises in the obtained data is one of the error factors. These errors are inevitable even with tools used for measurement such as noises filtering.

(3) Both FRF amplitude and phase measured at three points of the structure are evaluated resulting in a good conformity between the results obtained from the Wishart matrix model and experiments.

We realize that in practical use of this method, more complex systems should be studied to be able to pluralize Gaussian and Wishart random matrix models in calculating the uncertainty of the model. However, Current study may at least provide a basis for further investigations in this field.

Nomenclature

p , Σ	scalar and matrix parameters of the Wishart distribution	$\mathbf{A}(\omega)$	dynamic stiffness matrix
$\mathrm{Trace}(\bullet)$	sum of the diagonal elements of matrix	$\mathbf{f}(t)$	forcing vector
$\sim$	distributed as	pdf	probably density function
ω	excitation frequency	$\mathbb{R}$	space of real numbers
$\|\bullet\|_F$	Frobenius norm of a matrix	$\mathbb{R}^{n\times m}$	space $n\times m$ real positive definite matrices
FRF	frequency response function	$\mathbb{R}^{n\times n}$	space $n\times n$ real positive definite matrices
SFEM	Stochastic finite element method	$\Gamma_n(a)$	multivariate gamma function
$\hat{\mu}$	Gaussian mean matrix	$\otimes$	Kronecker product
$\mathbf{G}$	symbol for a system matrix	$\hat{\mathbf{U}}$, $\hat{\mathbf{V}}$	covariance matrix parameters of the Gaussian distribution

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