A Mathematical Model for Analysis of Upright Perforated Wave Absorbers

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Abstract
On the basis of the continuity equation and the Bernoulli equation in the steady form, a differential equation is developed to evaluate the successive water levels within compartments of an upright perforated wave absorber. Then the initial and boundary conditions are introduced and the differential equation is solved as an initial value problem. Finally the reflection coefficient from the wave absorber is calculated by establishing a balance between the rates of energy dissipated, the energy propagation and the energy reflected of the incident waves. The results of numerical model are verified by experimental tests with regular waves. A new non-dimensional parameter is introduced to characterise the hydraulic performance of the upright perforated wave absorber. It is shown that the rate of wave dissipation through the absorber decreases as this parameter increases.

Keywords: Upright Perforated Wave Absorbers, Wave Dissipation, Wave Reflection, Wave Flume, Wave Basin

1. Introduction

Reflection of wave energy from the boundaries of wave basins and wave flumes can result in the growth of a complex wave state and limit the functionality of such facilities. Wave absorbers are sometimes used to dissipate the wave energy at the end of a wave flume or along the rigid boundaries of a wave basin thereby reducing the reflected wave energy. Normally, the distance required to provide adequate dissipation of the wave energy by conventional beach type wave absorbers is at least one wave-length from the basin wall.

It is desirable that the coefficient of wave reflection from the absorber be less than 5%.

Achieving such a low percentage of wave reflection may be difficult, especially when only limited room exists for installation of the wave absorber.

The notion of using a perforated wall in front of a rigid barrier as an effective wave dissipater was originally proposed by Jarlan (1961), and his design has been used for the protection of some oil platforms in the Gulf of Mexico and in the North Sea. Since then there have been a number of studies of the hydraulic performance of perforated wall breakwaters on a theoretical and experimental basis.

The concept of employing rows of perforated sheets in front of a backed solid wall for absorbing wave energy in wave tanks has been introduced by Jamieson and Mansard (1987); and Jamieson et al. (1989). These wave attenuators are called upright perforated wave absorbers. Experience in the past decade has shown that...
this type of wave absorber could attenuate the incident wave energy more effectively if the porosity of perforated sheets is decreased toward the rear of the absorber (Jamieson et al., 1989).

When a wave impinges on an upright perforated wave absorber, part of its energy is reflected and the remainder passes through the perforated sheets. The energy of the incident wave is converted to the form of a series of jets, issuing from screens' perforations, which dissipate the wave energy through the generation of turbulence and eddies.

Generally, the wave reflection from an upright perforated wave absorber is due to the reflections from individual screens and reflection from the end wall. For a wave absorber with low porosity screens a considerable part of the incident wave energy is reflected from the first screen. On the other hand, for a wave absorber composed of high porosity screens the reflection is mainly due to the solid end wall.

A wave absorber of higher porosity screens is most effective in absorbing the energy of the steeper waves, whereas an absorber of lower porosity sheets is most effective in dissipation of the energy of waves of lower steepness (Jamieson et al., 1989). Therefore as Le Méhauté (1972) suggested a progressive wave absorber in which the screen porosity is decreased toward the rear of the absorber is more effective in attenuating the incident wave energy with minimal reflection from the absorber and for wide ranges of the incident wave steepness.

During the last three decades various theoretical solutions have been proposed for analysis of perforated wall breakwaters composed of one or two chambers and upright perforated wave absorbers (Terrett et al., 1968; Richey and Sollitt, 1970; Kondo, 1979; Fugazza and Natale, 1992; Twu and Lin, 1991).

The most difficult problem facing theoretical modelling of wave interaction with a porous media, such as an upright perforated wave absorber, is the existence of a non-linear term in the equation of motion. To solve this problem, it is customary assumed that the non linear term is replaceable by a linear term which gives the same average rate of wave dissipation. This idea, which is referred to the "Lorentz technique" in the literature, states that the realistic time-averaged turbulent flow resistance can be substituted by an equivalent time-averaged linear resistance (Terrett et al., 1968; Richey et al., 1970; Kondo, 1979; Fugazza et al., 1992; Lorentz, 1926; Jacobson, 1930; Proudman, 1953; Hayashi et al., 1966).

Among the proposed theoretical and numerical models for analysis of perforated wall breakwaters, only a few of them can be used for the case of wave absorbers composed of more than two perforated sheets. Twu and Lin (1991) derived a theoretical solution for analysis of upright perforated wave absorbers. They assumed that the flow inside the porous plate obeys the Darcy's law. However since in typical laboratory situations the Reynolds number is large and this law is not valid, their model can not be used for practical purposes.

Fugazza et al. (1992) gave theoretical analysis of a perforated breakwater composed of multiple rows of perforated plates. They used Lorentz technique in their analysis and replaced the non linear term of motion equation with an equivalent linear term. Therefore, their model is based on a linearism approximation. Faure (1992) provided a numerical model for evaluation of the wave reflection coefficient from an upright perforated wave absorber. However, the evaluation of reflection coefficient from the numerical model requires that the values of reflection and transmission coefficients and the values of phase shifts between reflected and transmitted waves and the incident wave be determined experimentally from the laboratory tests on single sheets.

On the basis of the motion and continuity equations associated with the Robertson and Lorentz techniques, Chegini and Wilkinson (1995) developed a differential equation for the motion of fluid within each compartment of an upright perforated wave absorber. After solving this equation, which is indeed
the equation of oscillation of the fluid within a series of connected tanks subjected to an extraneous forcing, the damping factor of screens and the elevation of water within compartments of the absorber were evaluated. Finally the reflection coefficient of the wave absorber was calculated by establishing a balance between the rates of energy dissipation and the energies of the incident and reflected waves. They performed a series of laboratory tests to measure the coefficients of wave reflection from the absorbers. They found a good agreement between the theoretical and experimental results.

Suh and Park (1995) developed analytical models for predicting wave reflection from a perforated-wall caisson breakwater. They used Galerkin-eigenfunction method to evaluate the reflection coefficient when waves are obliquely incident to breakwater at an arbitrary angle.

Isaacson et al. (2000) described a theoretical analysis for wave reflection from a breakwater consisting of a perforated front wall and a horizontal plate in it.

More recently, Li et al. (2003) studied the reflection of oblique waves by breakwaters that consist of a double-layered perforated wall and an impermeable back wall. The fluid domain was divided into three sub-domains and an eigenfunction method was applied in each of the domain. They compared the numerical results of the model with experimental data. Finally, they examined the major factors that affect the reflection coefficient.

2. Theoretical Development

2.1. Assumptions

Consider a number of perforated sheets aligned normally to the direction of wave propagation (fig.1). It is assumed that:

i) The flow between screens is in the form of a series of jets through each perforation, driven by the pressure differential across the screens.

\[ v \frac{\partial v}{\partial x} \gg \frac{\partial u}{\partial t} \]  

where:
- \( v \) jet velocity
- \( x \) horizontal coordinate
- \( u \) horizontal particle velocity of the wave
- \( t \) time

ii) The jet velocities are appreciably greater than the orbital velocities associated with the wave motion between the screens, i.e.

\[ \frac{\partial \eta}{\partial t} \gg \frac{q \partial \eta}{h \partial x} \]  

where:
- \( q \) flow per unit channel width through a perforated sheet
- \( \rho \) fluid density
- \( h \) water depth

Assumption (ii) implies that the convective acceleration through the screens \( (u \partial u/\partial x) \) is appreciably greater than the local acceleration within the wave tank \( (\partial u/\partial t) \). For relatively low amplitude and long waves the jet velocities discharging through screen perforations are large and this assumption is acceptable.

2.2. Continuity Equation

The flow per unit width through the nth screen, \( q_n \) can be expressed by:
\[ q_n = \alpha \int_{-h}^{0} V_n(z) \, dz \]  
(3)

where:
- \( n \) number of scree
- \( \alpha \) constant, Screen factor
- \( c_d \) discharge per unit width of the channel
- \( p \) screen porosity
- \( V_n(z) \) jet velocity at level \( z \) across the \( n \)th screen

The continuity equation dictates that:

\[ q_n = \frac{d}{dt} \int_{x_n}^{x_{n+1}} \eta_n \, dx + q_{n+1} \]  
(4)

where:
- \( \eta_n \) water level in the \( n \)th compartment
- \( t \) time

Combining equation (4) with assumption (iii) yields (fig. 2):

\[ q_n = \Delta \eta_n + q_{n+1} \]  
(5)

Fig. 2. Definition sketch for applying the continuity equation

2.3. Equation of Motion

Using the assumption (ii), the Bernoulli equation in the steady form may be written as follows:

\[ \frac{1}{2} \rho V_n^2(z) = [P_{n-1}(z) - P_n(z)] \]  
(6)

or:

\[ V_n(z) = \sqrt{2\rho [P_{n-1}(z) - P_n(z)]} \]  
(7)

Assuming the differential pressure across the \( n \)th screen has resulted from the differential hydrostatic pressures in \((n-1)\)th and \(n\)th compartments, plus the effect of jet velocity across the \((n-1)\)th screen, we can write:

\[ P_{n-1}(z) - P_n(z) = \rho g(\eta_{n-1} - \eta_n) + \beta \frac{\rho}{2} V_n^{-1} |V_{n-1}| \]  
(8)

where \( \beta \) is an empirically determined component of dynamic pressure differential.

The first term on the right hand side of equation (8) is the difference between hydrostatic pressures at either side of the \( n \)th sheet. Whereas, the second term is the dynamic pressure caused by the wave motion when it passes through perforations of the \((n-1)\)th screen. Thus assuming \( \beta = 0 \) it follows that the pressures at either side of the screen are distributed hydrostatically, which is an appropriate assumption for small amplitude and long waves. Indeed, \( \beta \) is a factor which allows for dynamic effects omitted from the steady form of the Bernoulli equation.

2.4. Derivation of Differential Equation

Eliminating \( V_n(z) \) between (3) and (6) yields:

\[ q_n = \alpha \int_{-h}^{0} \frac{\rho}{2} [P_{n-1}(z) - P_n(z)]^{1/2} \, dz \]  
(9)

Combining equations (5), (8) and (9) it may be shown that:

\[ \frac{\partial \eta_n}{\partial t} = \frac{\alpha h}{S} \left\{ \left[2g(\eta_{n-1} - \eta_n) + \beta V_n^{-1} |V_{n-1}| \right]^{1/2} - \left[2g(\eta_n - \eta_{n+1}) + \beta V_n |V_n| \right]^{1/2} \} \]  
(10)

Where \( g \) is the acceleration due to gravity. Equation (10) may be non-dimensionalized and normalized by employing the following parameters:

\[ N = \frac{\eta}{h_i} \]  
(11)
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\[ \tau = \frac{t}{T} \]

\[ \lambda = \frac{gT^2}{H_i} \]

\[ U = \frac{VT}{H_i} \]

\[ Q = \frac{qT}{Sh} \]

Where \( H_i \) and \( T \) are the incident wave height and the wave period, respectively.

The normalized form of equation (10) is given by:

\[ N_n^{t+\Delta t} = N_n^t + F \Delta \tau \left[ \frac{(N_{n-1}^t - N_n^t)^{1/2} - (N_{n+1}^t - N_n^t)^{1/2}}{2} \right] \]

where:

\( \Delta \tau = i F. \Delta \tau \left[ (N_{n-1}^t - N_n^t)^{1/2} - j(N_n^t - N_{n+1}^t)^{1/2} \right] \)

(15)

\( \Delta \tau = i F. \Delta \tau \left[ (N_{n-1}^t - N_n^t)^{1/2} - j(N_n^t - N_{n+1}^t)^{1/2} \right] \)

where:

- \( i = 1, j = 1 \) for \( N_{n-1}^t \geq N_n^t \geq N_{n+1}^t \)
- \( i = -1, j = 1 \) for \( N_{n-1}^t \leq N_n^t \leq N_{n+1}^t \)
- \( i = -1, j = -1 \) for \( N_{n-1}^t < N_n^t < N_{n+1}^t \)

(figure 3: a, b, c & d, respectively)

Fig 3. Different directions of inflow and outflow discharges for a compartment during a wave period

2.5. Initial and Boundary Conditions

An initial condition which satisfies all the relevant physical relationships is given by:

\[ N_n = 0 \quad \text{at} \quad t = 0 \quad \text{for} \quad n = 1, 2, 3, \ldots \quad (16) \]

The appropriate boundary conditions in the front and at the end of the absorber are set as:

\[ N_n = \frac{a_0}{H_i} \sin(\sigma t) \quad \text{for} \quad n = 1 \]

\[ V_n = 0 \quad \text{for} \quad n = n_s + 1 \quad (17) \]

where:

\[ a_0 = a_i + a_r = (1 + K_r)a_i \]

\( a_r \) reflected wave amplitude
2.6. Evaluation of Reflection Coefficients

The perforations may be simulated as orifices that cause the loss of wave energy through the production of jet turbulence and the rate of energy loss by this means is given by:

\[ \frac{dE_g}{dt} = \rho \sum_{n=1}^{N} \frac{1}{2} \tilde{V}_j^3 A_{jet} \]  \hspace{1cm} (18)

over the depth, \( h \), and unit breadth of the perforated sheet.

where:
\( \rho \) the fluid density
\( A_{jet} \) area of the jet
\( \tilde{V}_j \) time averaged jet velocity issued from a screen

The average rate of energy dissipation for each sheet is found by integration, with respect to time, over one wave cycle divided by the wave period. The total rate of energy dissipation, \( D \), is obtained by summing up the rate of energy dissipation of all sheets.

Thus:

\[ D = \sum_{n=1}^{N} \frac{1}{T} \int_{t}^{t+T} \frac{1}{2} \tilde{V}_j^3 A_{jet} dt \]  \hspace{1cm} (19)

Where \( A_{jet} \) is calculated for unit width of the channel. The average rate of energy propagation (or incident wave power) per unit crest width is:

\[ P_i = \frac{1}{2} \gamma c_g a_i^2 \]  \hspace{1cm} (20)

where \( c_g \) is the wave group velocity.

Assuming that no energy is transmitted beyond the wave absorber, it follows that:

\[ P_r = P_i - D \]  \hspace{1cm} (21)

Substituting (18) and (19) in (20) yields:

\[ P_r = \frac{1}{2} \rho g c_s a_i^2 - \sum_{n=1}^{N} \frac{1}{T} \int_{t}^{t+T} \sum_{j=1}^{\infty} \tilde{V}_j^3 A_{jet} dt \]  \hspace{1cm} (22)

The averaged rate of energy reflected is:

\[ P_r = \frac{1}{2} \rho g c_s a_i^2 \]  \hspace{1cm} (23)

Combination of equations (21) and (22) gives:

\[ K_r = [1 - \frac{2}{c_s a_i^2} \sum_{n=1}^{N} \frac{1}{T} \int_{t}^{t+T} \sum_{j=1}^{\infty} \tilde{V}_j^3 A_{jet} dt]^{1/2} \]  \hspace{1cm} (24)

2.7. Numerical Solution

Equation (11) can be solved numerically as an initial value problem, with equations (16) and (17) giving the initial condition and the boundary conditions. Employing a finite difference method the value of \( N_n \) at each time step can be evaluated from the values of \( N_{n-1} \) at the previous time step.

A programme was written to solve the above equation numerically. Water levels within compartments of an upright perforated wave absorber were calculated as a function of time until periodic solutions evolved. Reflection coefficients were evaluated by using equation (24).

3. The Dissipation Parameter

The rate of wave dissipation and reflection due to the upright perforated wave absorbers are functions of the wave characteristics (\( H_0, h, L_0 \)) and the absorber specifications (\( c_d, p, S, n_s \)). Considering equation (13) it can be seen that for a wave absorber made of definite number of perforated sheets, the effects of all of these variables are incorporated in the non-dimensional parameter \( \Omega \) which is termed
the dissipation parameter.

Transmission of wave energy between compartments of an upright wave absorber increases with increase in the value of the dissipation parameter. Figure (4) depicts the variations of surface level fluctuation within compartments of an upright perforated wave absorber composed of five screens as a function of the dissipation parameter. This figure is shown for the case of $\beta = 0$ and $(K_r)_{initial} = 0$.

**4. Experimental Studies**

A set of experiments was carried out in the Water Research Laboratory, the University of New South Wales (WRL) to measure the reflection coefficients and wave profiles within compartments of upright perforated wave absorbers (Chegini and Wilkinson, 1995). The laboratory tests were carried out in the deeper section of the 0.9 m wide, 1.75 m deep, 50 m long wave flume of WRL. This wave flume was equipped with a 40 kw hydraulic piston type wave maker capable of generating waves up to 30 cm in height and with periods of between 0.5 and 3 seconds. The reflection coefficients were measured for waves of steepness ratios ranging from 0.04 to 0.1.

All the tests were performed with water depth of 0.8m. The specifications of the absorbers and the incident wave characteristics are tabulated in Table (1). The diameters of screen perforations for plates of 40 and 62 percent porosity were 6.35 and 7.94 mm, respectively. Water surface elevations in front of the wave absorbers were measured using two fixed capacitive wire wave probes located at distances of 150 cm and 190 cm from the first plate of the attenuator. Surface elevations within compartments of the absorbers were measured by using wave probes installed in the middle of each compartment. The reflection coefficients were calculated from the "two probes technique" reported by Thornton and Calhoun (1972) [19].

**Table 1. Specifications of the performed tests on upright perforated wave absorbers**

<table>
<thead>
<tr>
<th>No. of Screens</th>
<th>$p$ (%)</th>
<th>$S$ (m)</th>
<th>$T$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>40 for all sheets</td>
<td>0.30</td>
<td>1.25, 1.35, 1.50, 1.75, 2.25 &amp; 2.50</td>
</tr>
<tr>
<td>3</td>
<td>62 for all sheets</td>
<td>0.30</td>
<td>1.25, 1.50, 1.75, 2.25 &amp; 2.50</td>
</tr>
<tr>
<td>4</td>
<td>40 for all sheets</td>
<td>0.30</td>
<td>2.25 &amp; 2.50</td>
</tr>
<tr>
<td>4</td>
<td>62 for all sheets</td>
<td>0.30</td>
<td>2.25 &amp; 2.50</td>
</tr>
</tbody>
</table>

**5. Experimental and Numerical Results**

The experiments show that the dissipation parameter, $\Omega$, is the most significant parameter characterising the hydraulic performance of upright perforated wave absorbers. The rate of incident wave energy dissipation decreases with increasing values of dissipation parameter. Also, the reflection coefficient from the absorber increases as $\Omega$ increases.

Figures (5) and (6) indicate the variations of dissipation coefficients, $K_d$, and reflection coefficients, $K_r$, as a function of $\Omega$ parameter for wave absorbers composed of three and four perforated plates. The discharge coefficients of screen perforations were evaluated from the following empirical formula from Dailey et al., 1966 (also used by Mei et al., 1974):

$$C_d = 0.6 + 0.4 (A_p/A)^{1/2}$$  \hspace{1cm} (25)

where $A_p$ and $A$ are the perforated and the total area of the screen, respectively.

Usually in the design of upright perforated wave absorbers, the ranges of values of wave
characteristics \((H, T, h)\) are given. Therefore, by using \(\Omega\) parameter it is possible to select the absorber specifications \((P, S, n_s)\) so that the reflection coefficient is restricted to a certain value. Figures (5) and (6) depict the results of hydraulic model in comparison with the experimental results for relatively long waves. These figures indicate that when \(S/L \leq 0.1\) and \(h/L \leq 0.15\), the reflection coefficients can be well predicted from this model. In the theoretical development of the hydraulic model, it was assumed that the local acceleration term is ignorable in comparison with the convective acceleration term.

6. Conclusion

Based on application of the continuity and the Bernoulli equations associated with the linearised
water waves theory, a mathematical model was developed for prediction of reflection coefficients from upright perforated wave absorbers.

The results of laboratory tests were presented for reflection coefficients and wave profiles within compartments of upright perforated wave absorbers composed of three and four screens. The experiments indicated that:

i) The dissipation parameter, defined in equation (13), was the most significant parameter characterising the hydraulic performance of upright perforated wave absorbers

ii) The rate of dissipation of incident wave energy by an upright perforated wave absorber can be described as a function of the perforated wave dissipater parameter, $\Omega$. This rate decreases as $\Omega$ increases.

iii) Reflection coefficient increases with increasing the dissipation parameter.

iv) For wave absorbers composed of medium and high porosity screens used in the experiments, $K_r$ decreases as $H_i/L$ increases.

v) For a constant value of $H_i/L$, reflection coefficient increases with increasing screen porosity.

vi) Increasing the number of screens can be generally yielded to more dissipation of absorbers.

Good agreement was found between the results of hydraulic model and the experiments, when $h/L \leq 0.15$ and $S/L \leq 0.1$.

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